EE 505
Lecture 7

Windowing

Spectral Performance of Data Converters

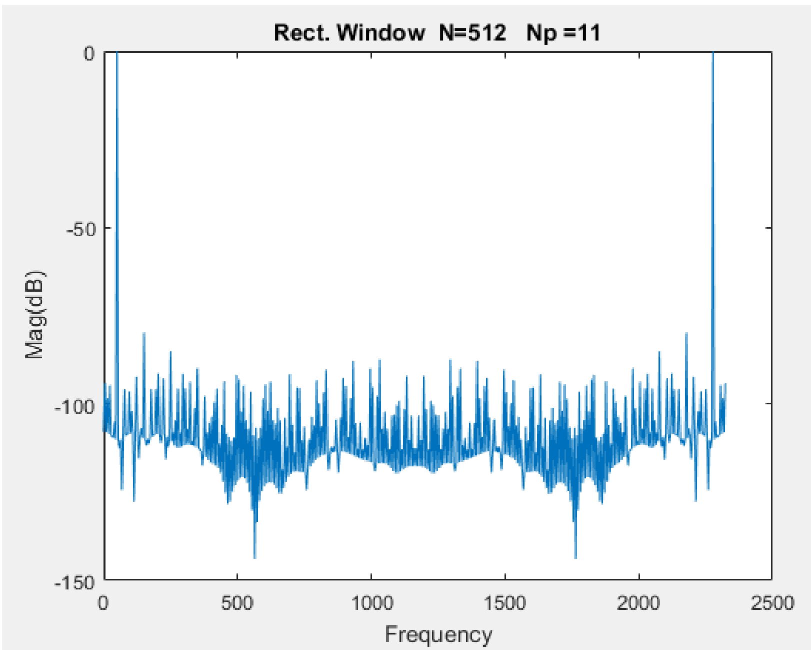
- Time Quantization
- Amplitude Quantization

Clock Jitter

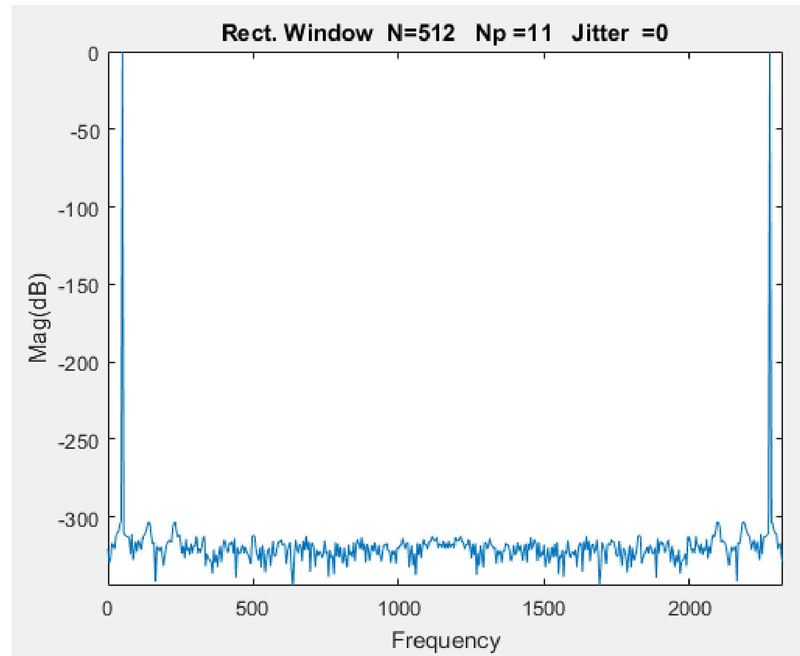
Statistical Circuit Modeling

Review from last lecture

MatLab comparison: 512 Samples with Standard Sweep



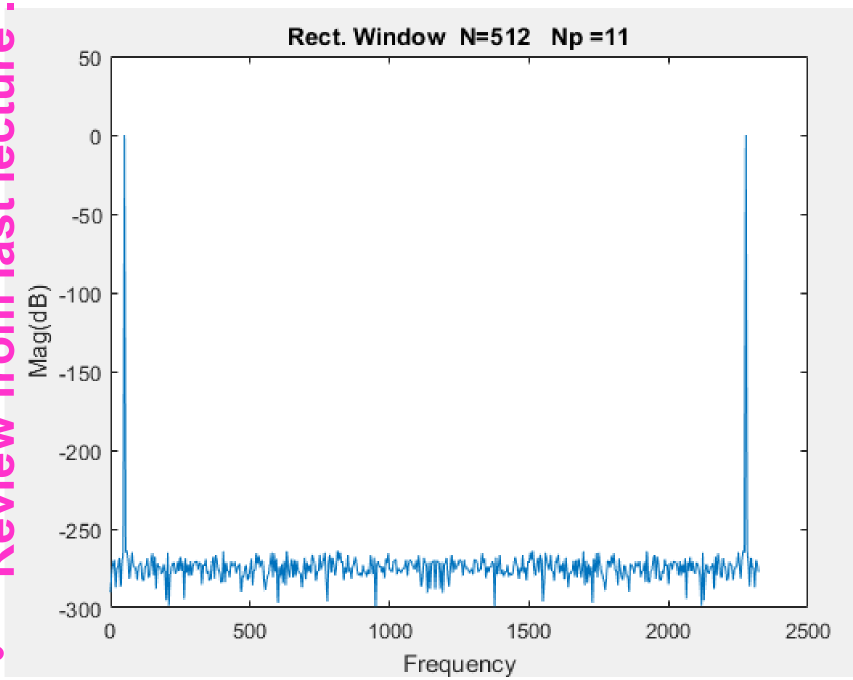
Spectre Results



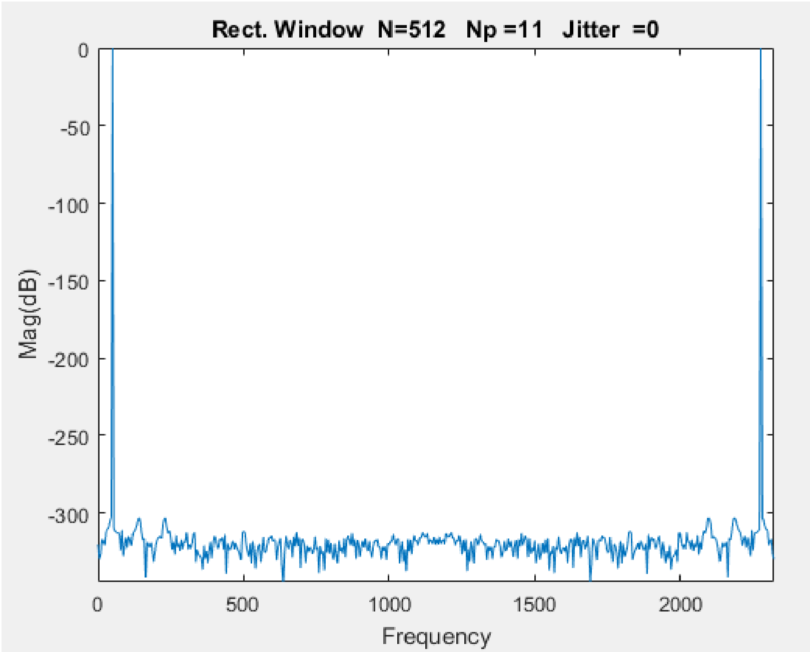
MatLab Results

MatLab comparison: 512 Samples with Strobe Period Sweep

Review from last lecture



Spectre Results



MatLab Results

Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing

Windowing - a strategy to address the problem of requiring precisely an integral number of periods to use the DFT for Spectral analysis?

- Windowing is sometimes used
- Windowing is sometimes misused

Recall

Example

WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

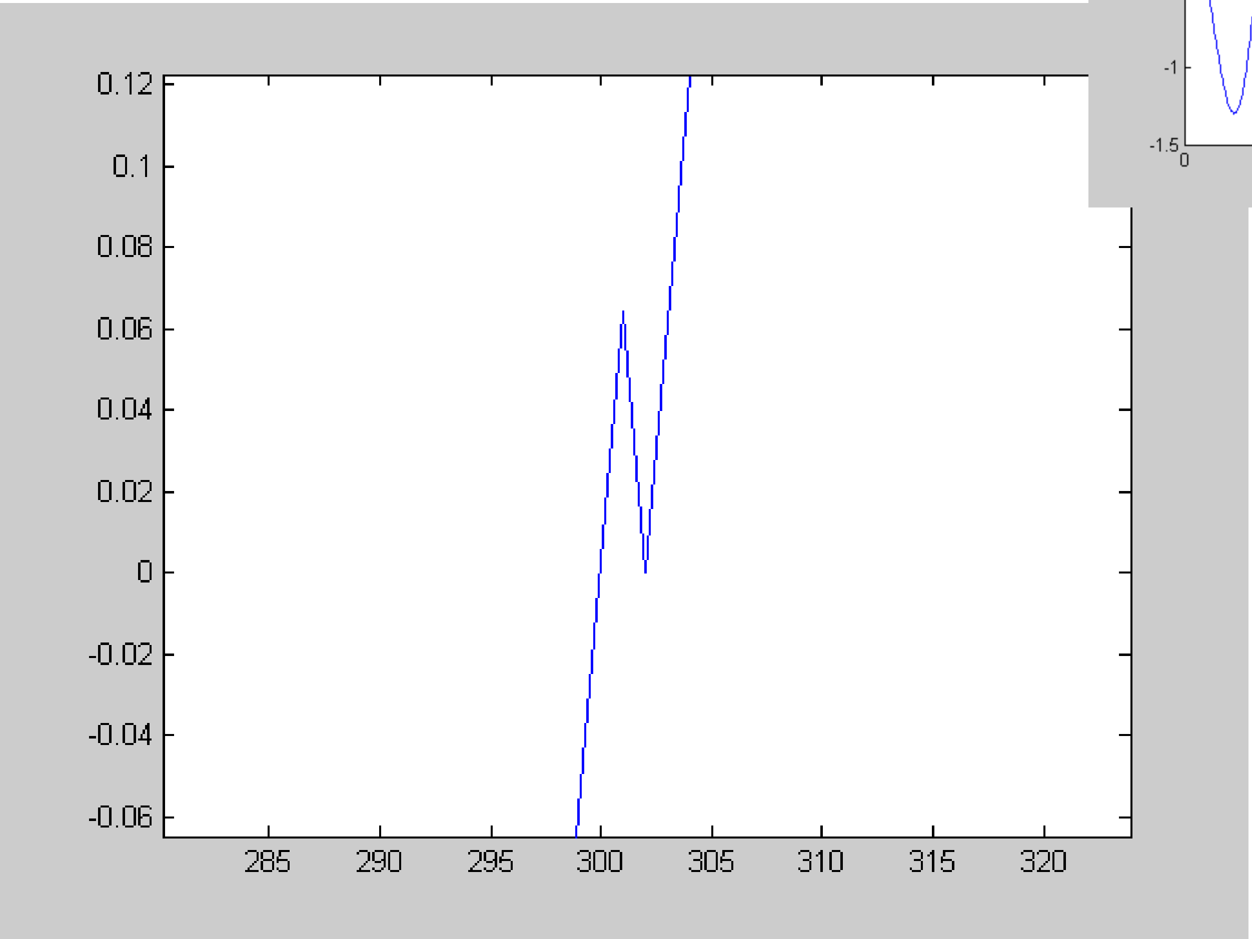
$$\omega = 2\pi f_{SIG}$$

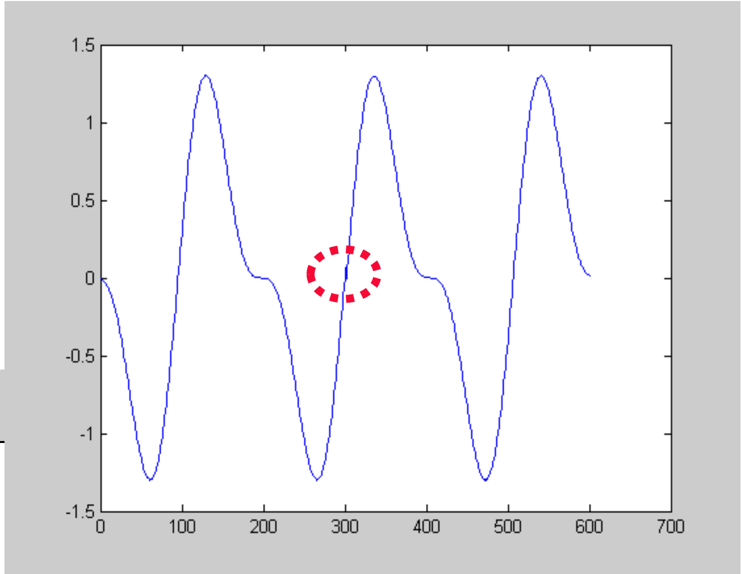
Consider N_P=20.01 N=4096

Deviation from hypothesis is .05% of the sampling window

Recall

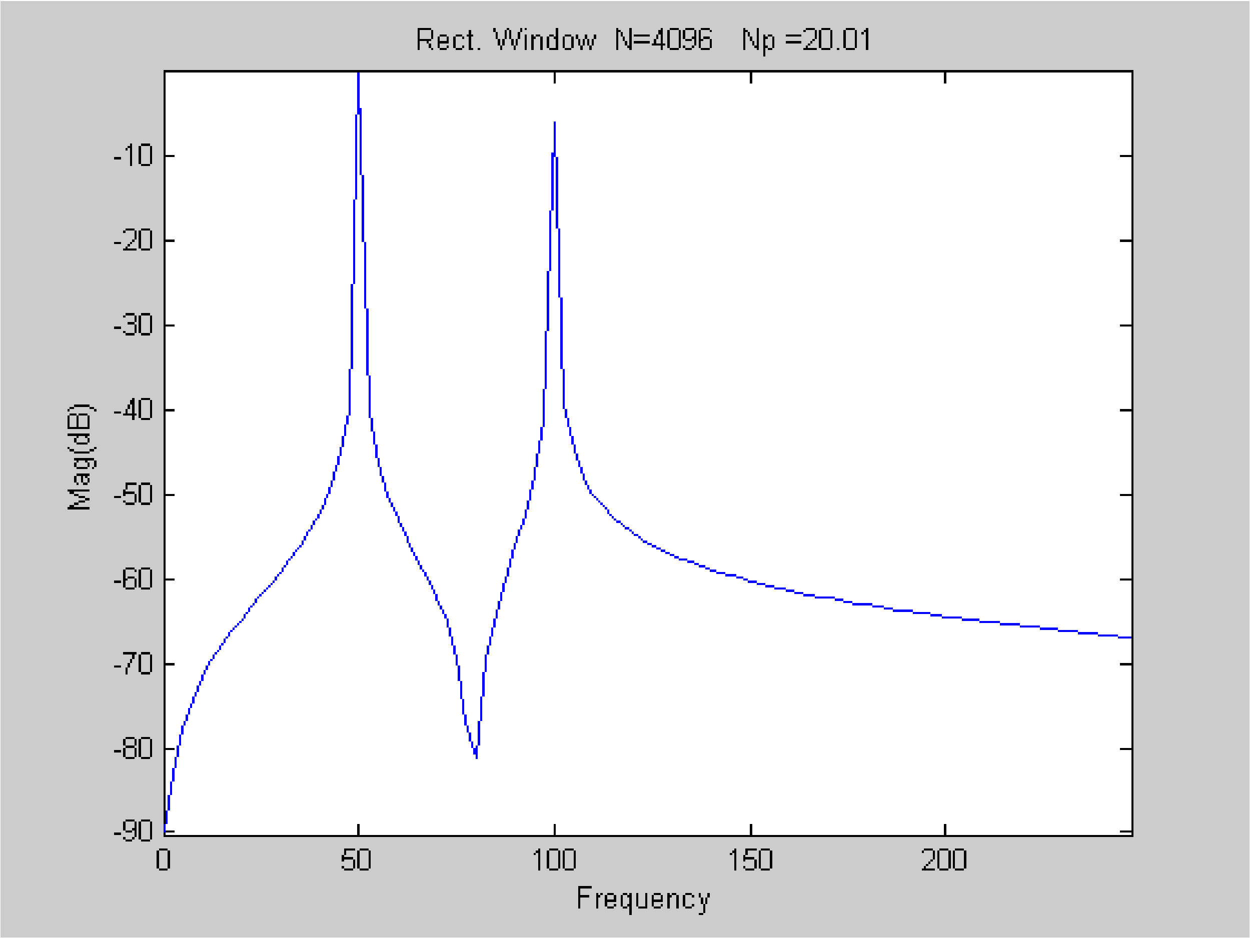
Input Waveform





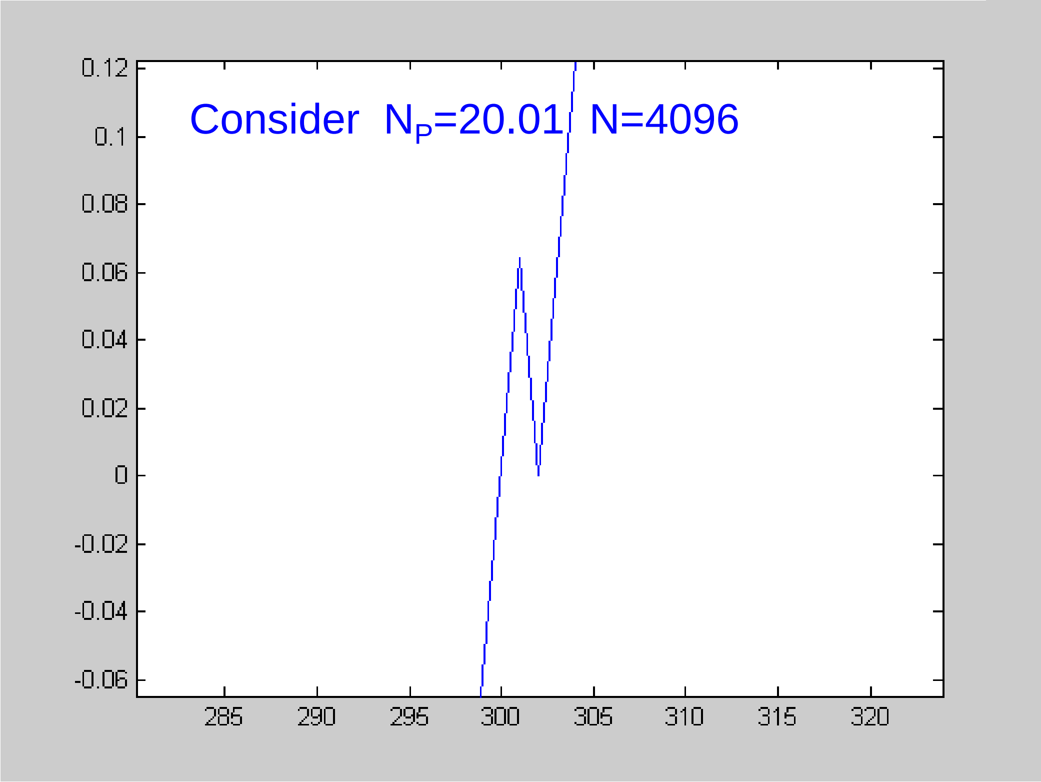
Recall

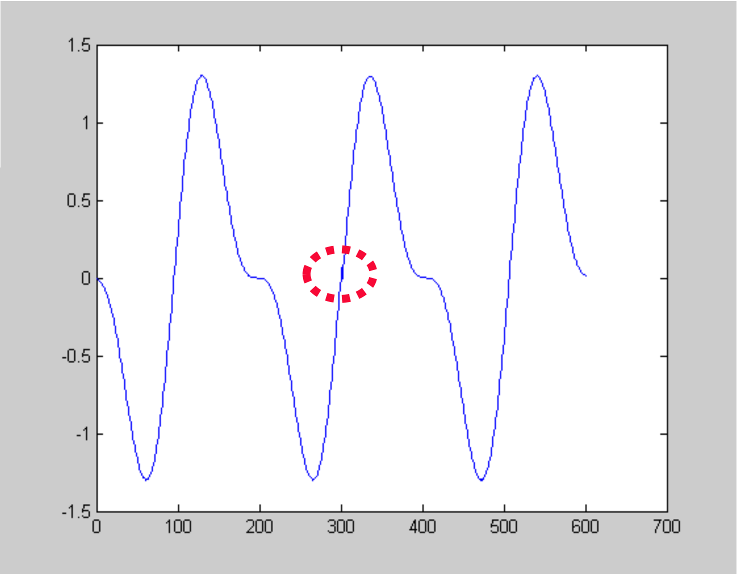
Spectral Response with Non-Coherent Sampling



(zoomed in around fundamental)

Recall





Even with N_P=20.001 had significant degradation

Extremely small discontinuity associated with non-coherent sampling causes Significant degradations in spectral response if DFT (and Theorem) used

Windowing

Windowing is the weighting of the time domain function to maintain continuity at the end points of the sample window

Well-studied window functions:

- Rectangular (also with appended zeros)
- Triangular
- Hamming
- Hanning
- Blackman

Rectangular Window

Sometimes termed a boxcar window

Uniform weight

Can append zeros

Without appending zeros equivalent to no window

Rectangular Window

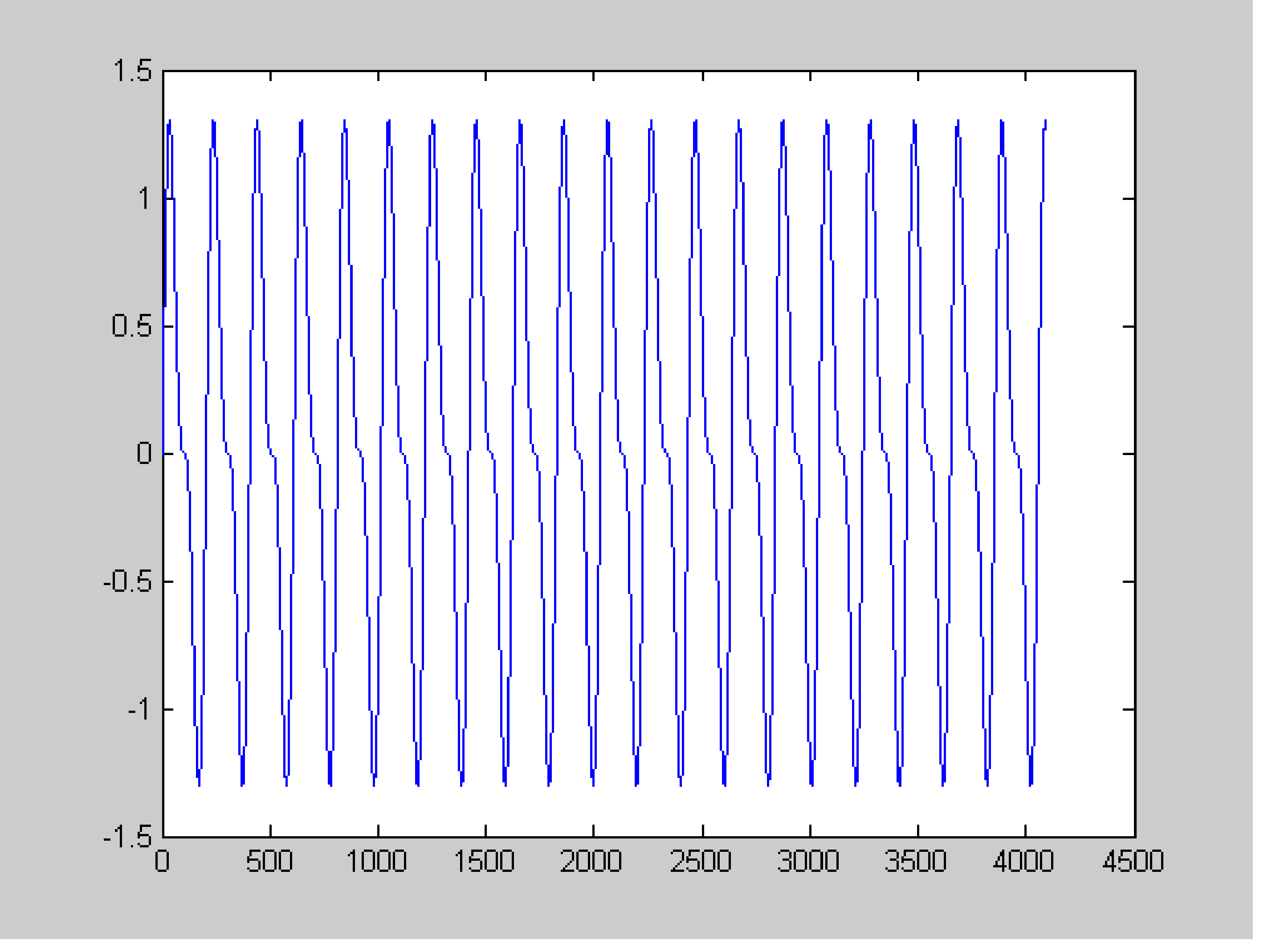
Assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

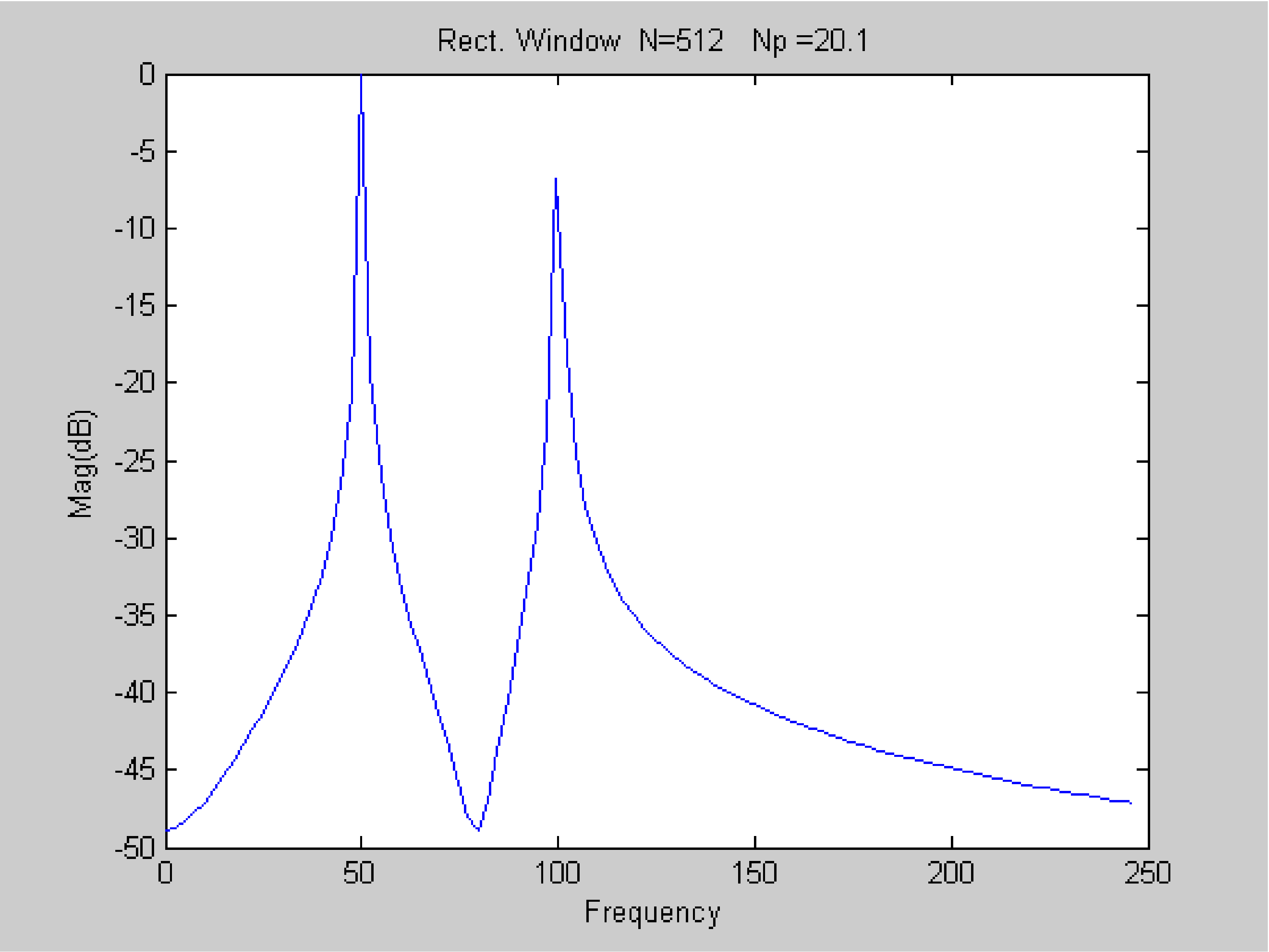
$$\omega = 2\pi f_{SIG}$$

Consider N_P=20.1 N=512

Rectangular Window



Spectral Response with Non-coherent sampling



(zoomed in around fundamental)

Rectangular Window

Columns 1 through 7

-48.8444	-48.7188	-48.3569	-47.7963	-47.0835	-46.2613	-45.3620

Columns 8 through 14

-44.4065	-43.4052	-42.3602	-41.2670	-40.1146	-38.8851	-37.5520

Columns 15 through 21

-36.0756	-34.3940	-32.4043	-29.9158	-26.5087	-20.9064	-0.1352

Columns 22 through 28

-19.3242	-25.9731	-29.8688	-32.7423	-35.1205	-37.2500	-39.2831

Columns 29 through 35

-41.3375	-43.5152	-45.8626	-48.0945	-48.8606	-46.9417	-43.7344

Rectangular Window

Columns 1 through 7

-48.8444	-48.7188	-48.3569	-47.7963	-47.0835	-46.2613	-45.3620

Columns 8 through 14

-44.4065	-43.4052	-42.3602	-41.2670	-40.1146	-38.8851	-37.5520

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-36.0756	-34.3940	-32.4043	-29.9158	-26.5087	-20.9064	-0.1352

Columns 22 through 28

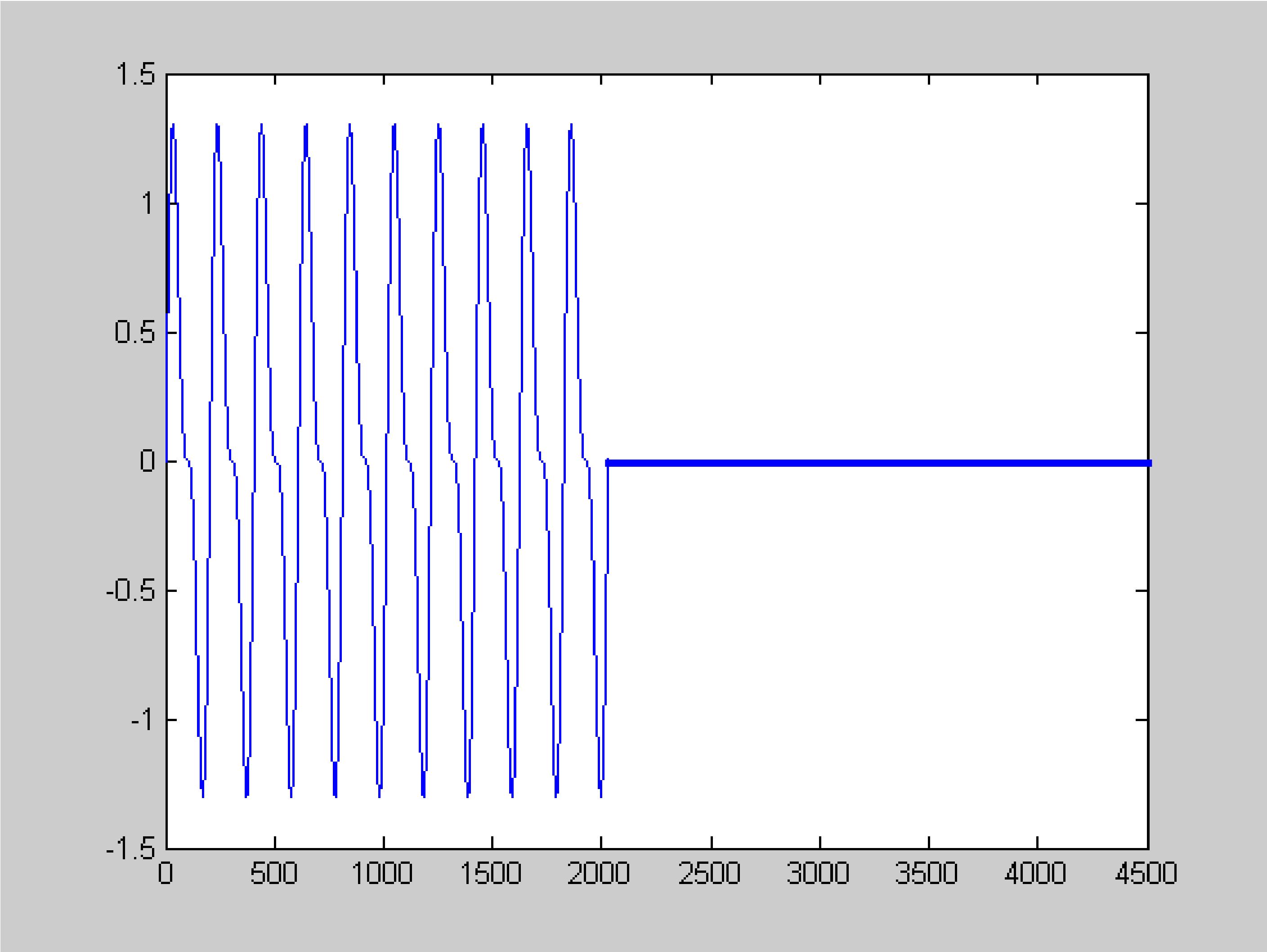
-19.3242	-25.9731	-29.8688	-32.7423	-35.1205	-37.2500	-39.2831

Columns 29 through 35

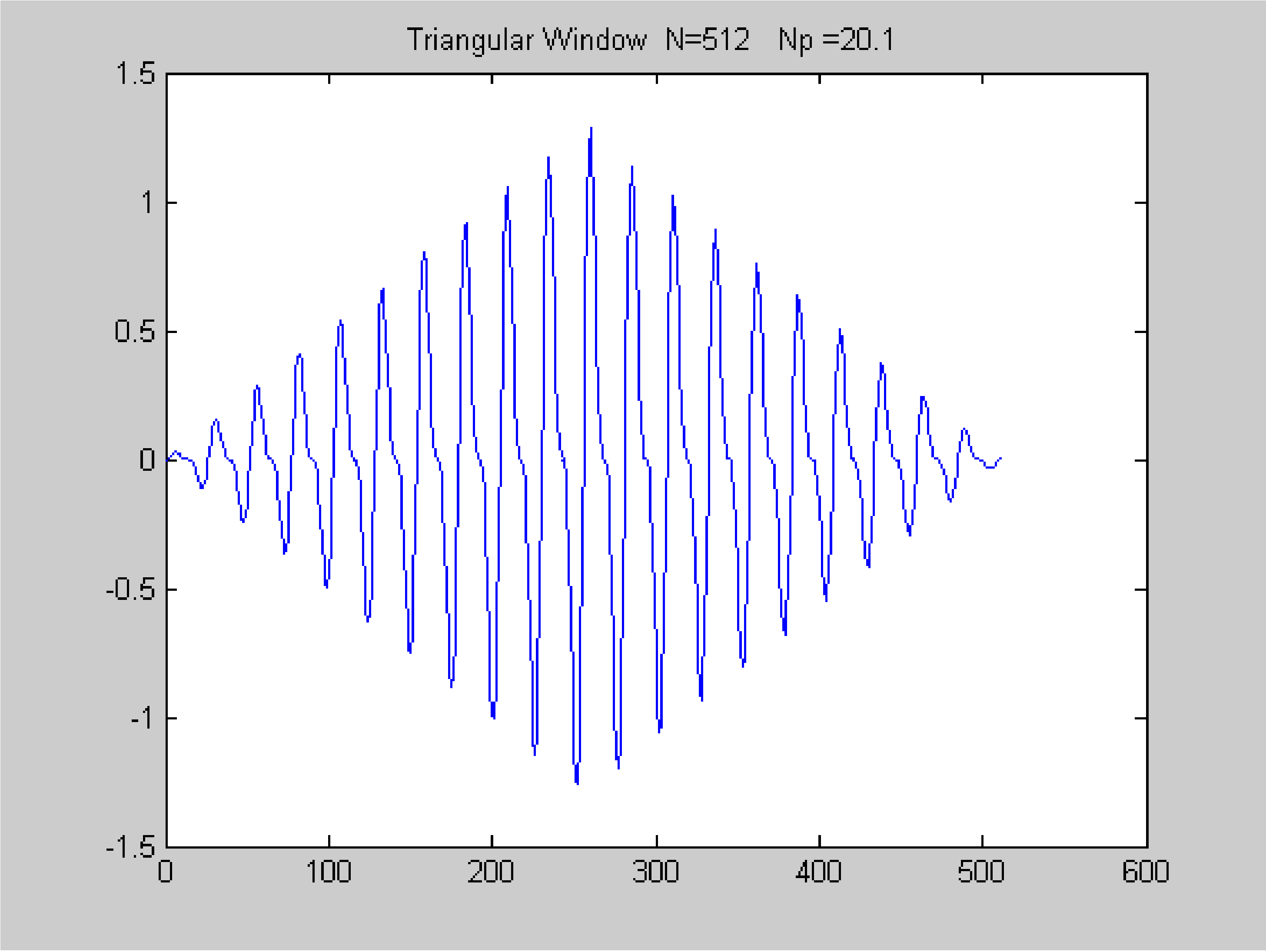
-41.3375	-43.5152	-45.8626	-48.0945	-48.8606	-46.9417	-43.7344

Energy spread over several frequency components

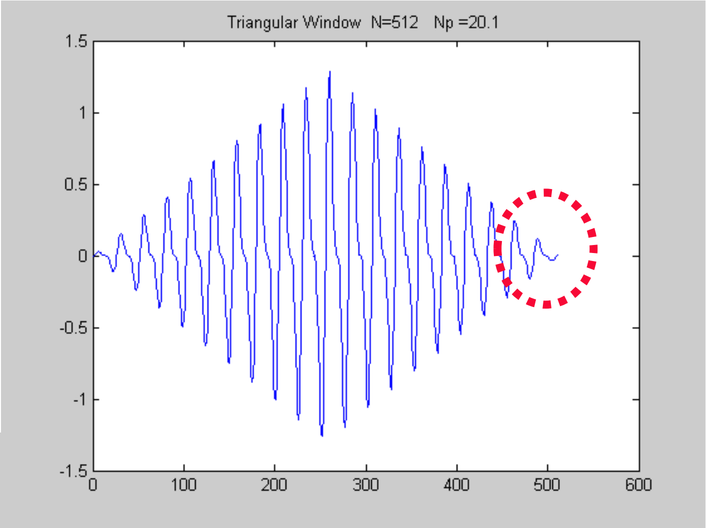
Rectangular Window (with appended zeros)

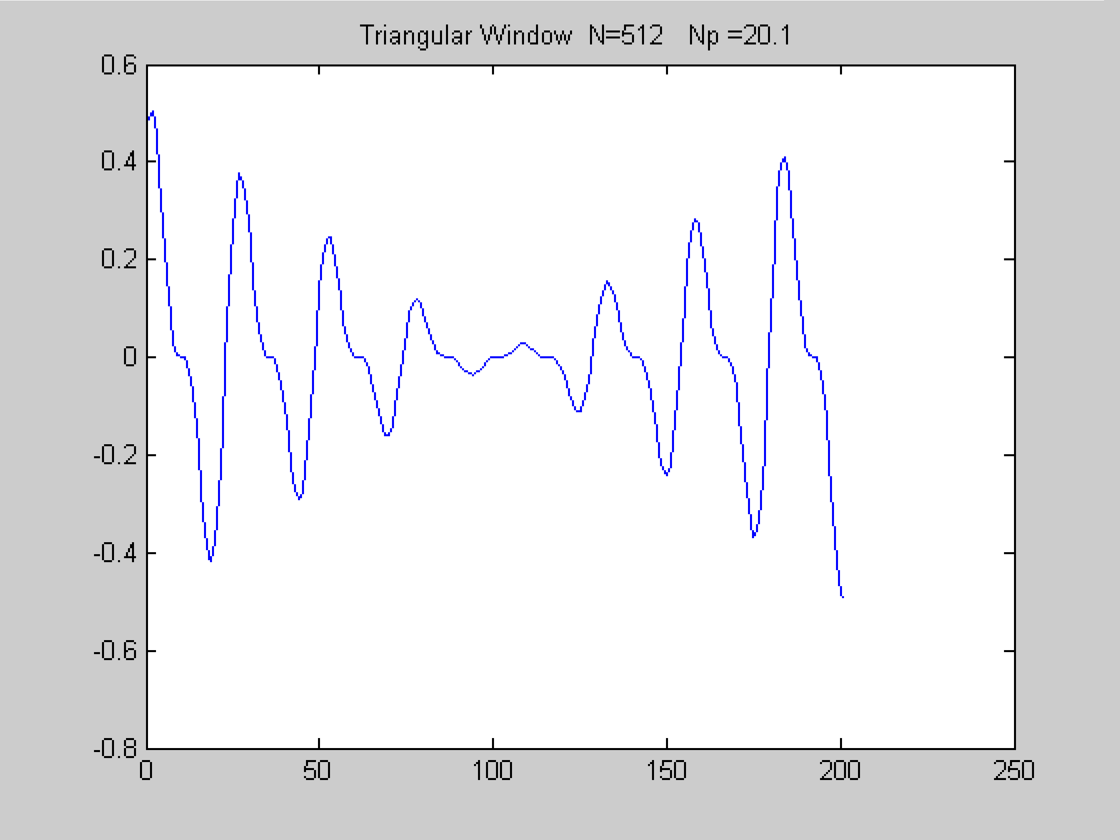


Triangular Window

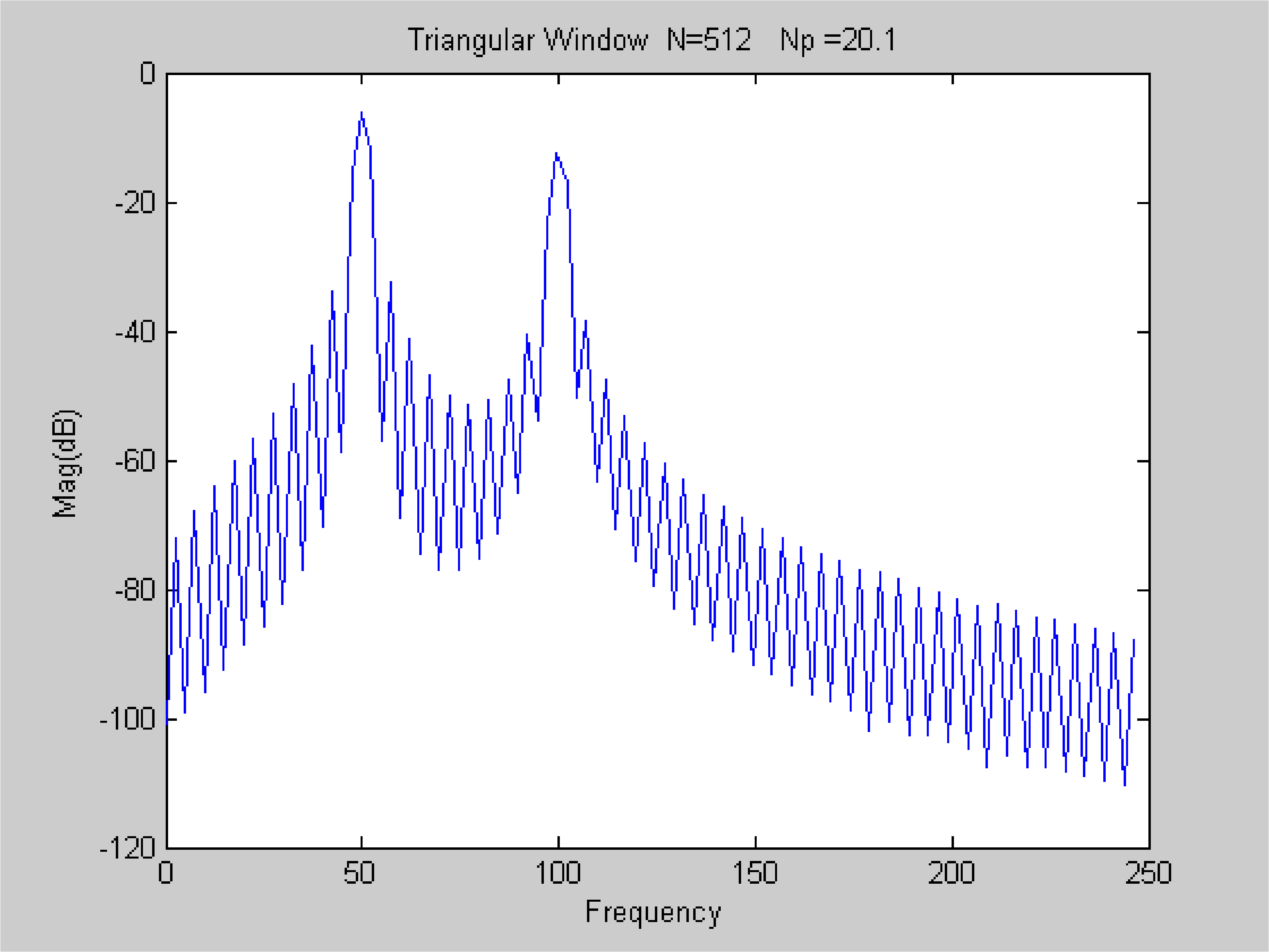


Triangular Window



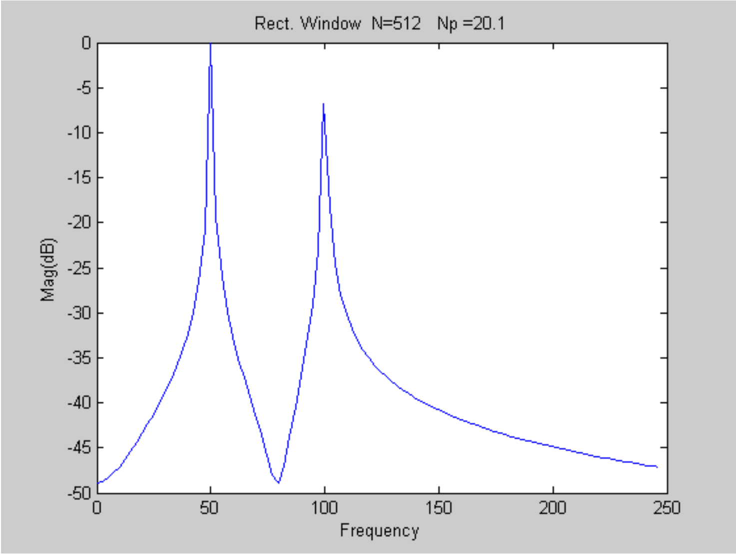


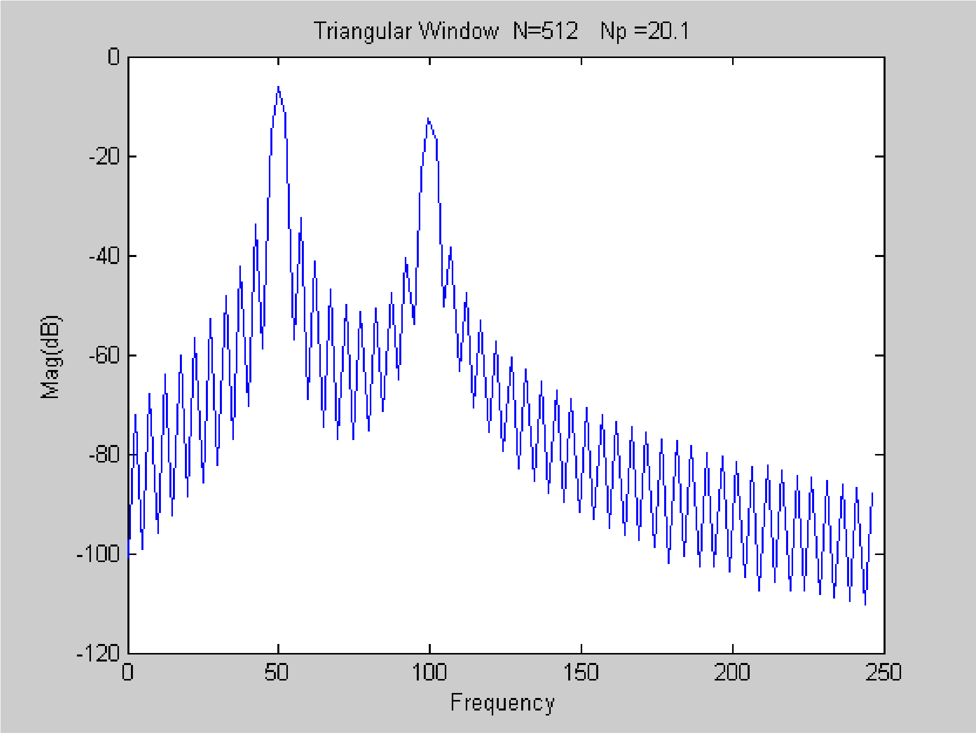
Spectral Response with Non-Coherent Sampling and Windowing



(zoomed in around fundamental)

Triangular Window





Triangular Window

Columns 1 through 7

-100.8530	-72.0528	-99.1401	-68.0110	-95.8741	-63.9944	-92.5170

Columns 8 through 14

-60.3216	-88.7000	-56.7717	-85.8679	-52.8256	-82.1689	-48.3134

Columns 15 through 21

-77.0594	-42.4247	-70.3128	-33.7318	-58.8762	-15.7333	-6.0918

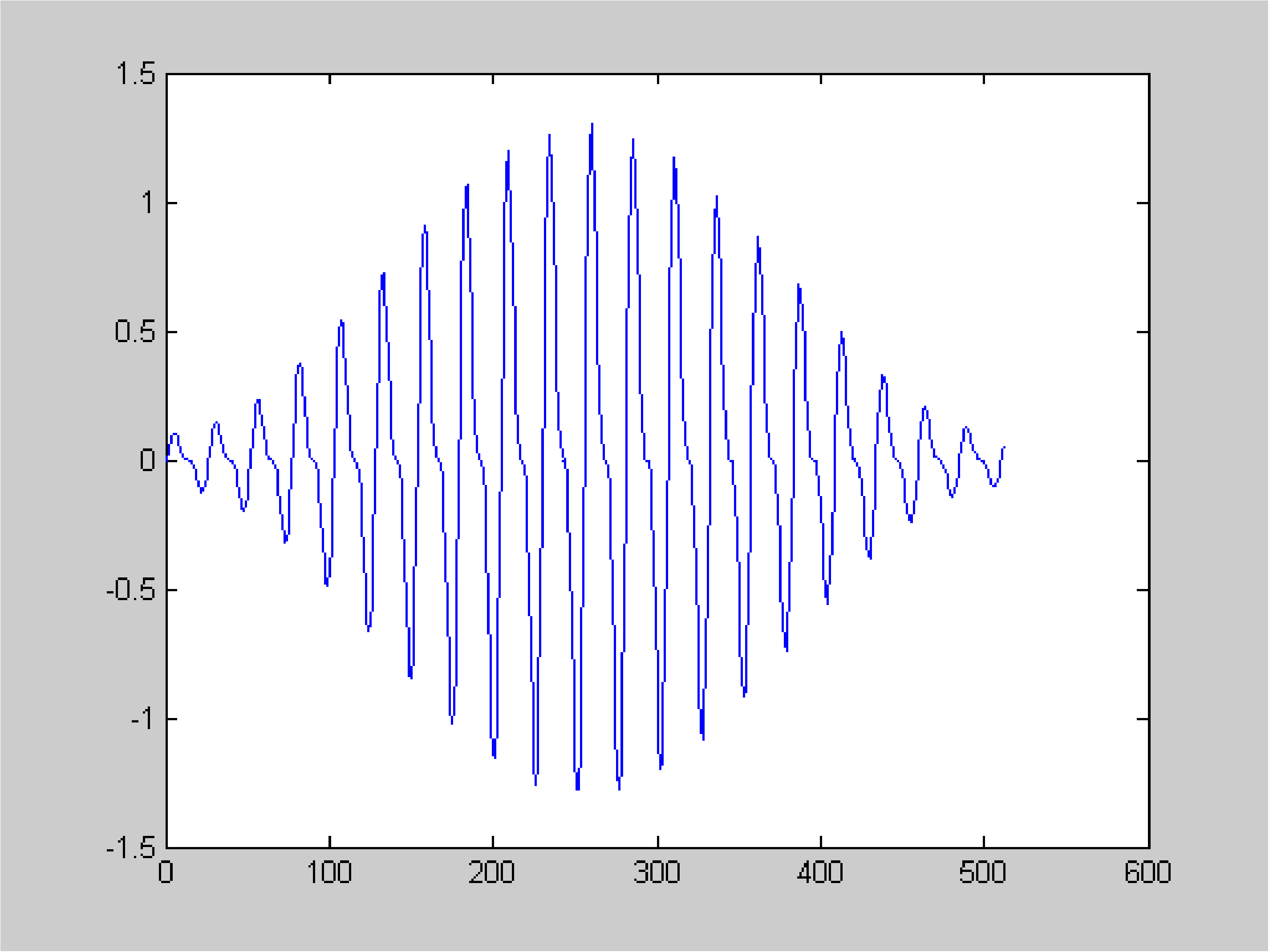
Columns 22 through 28

-12.2463	-57.0917	-32.5077	-68.9492	-41.3993	-74.6234	-46.8037

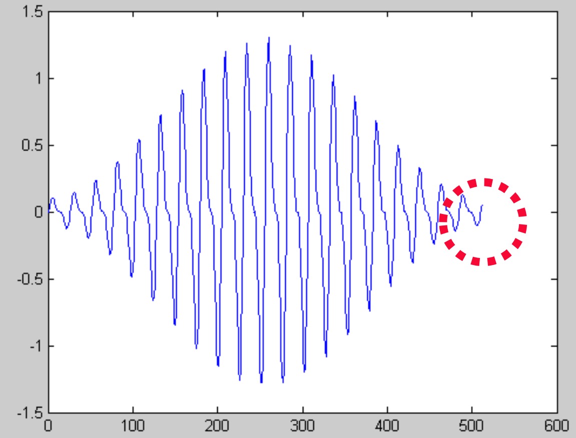
Columns 29 through 35

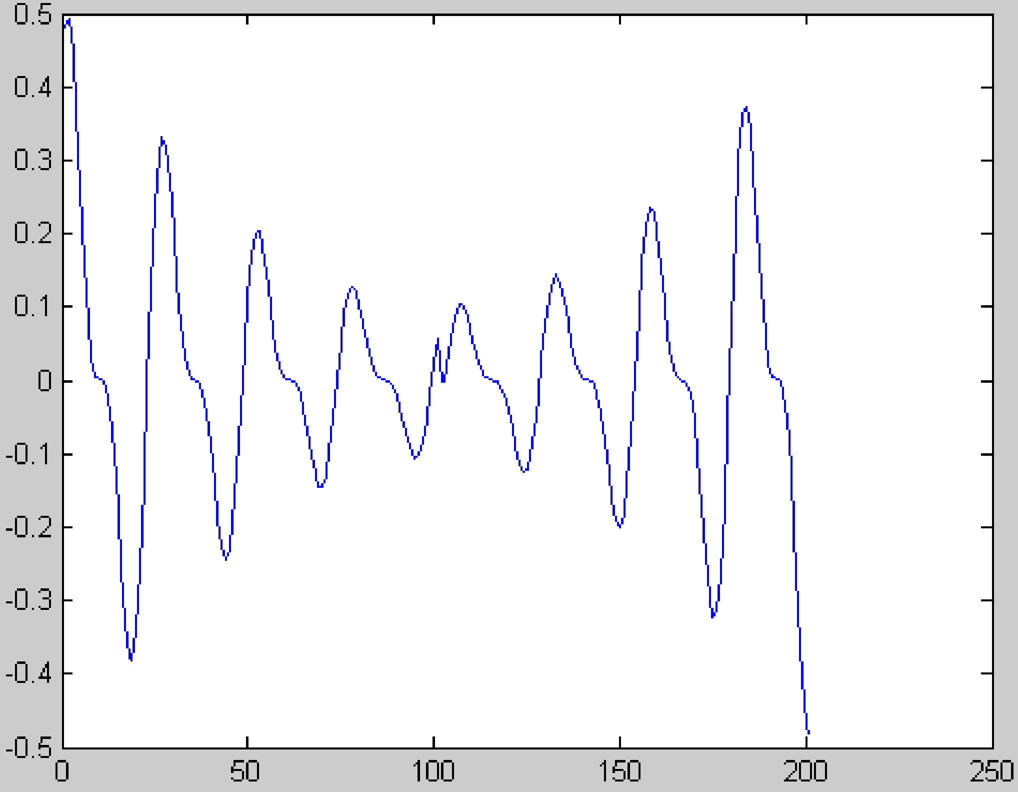
-77.0686	-50.1054	-77.0980	-51.5317	-75.1218	-50.8522	-71.2410

Hamming Window

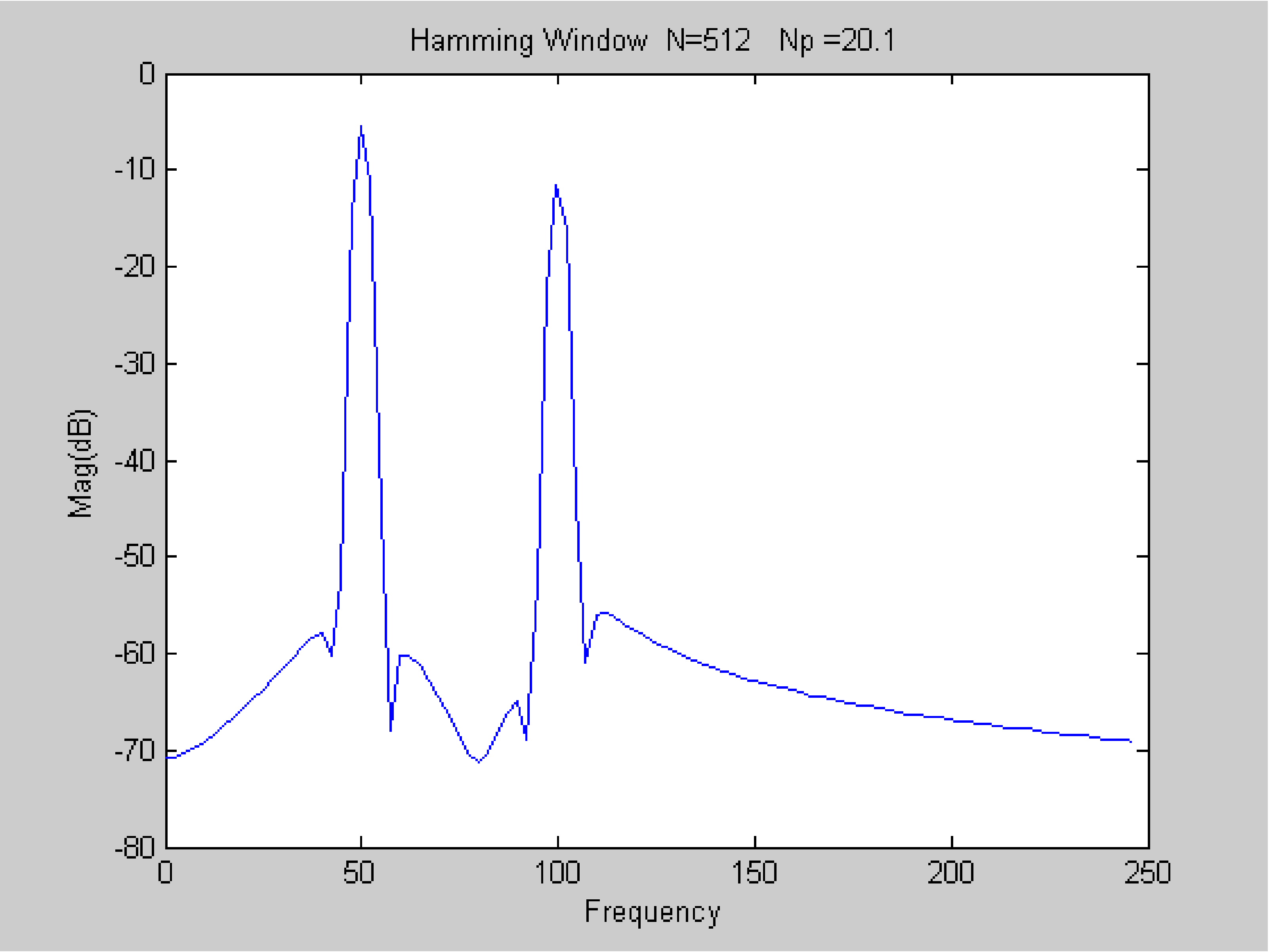


Hamming Window



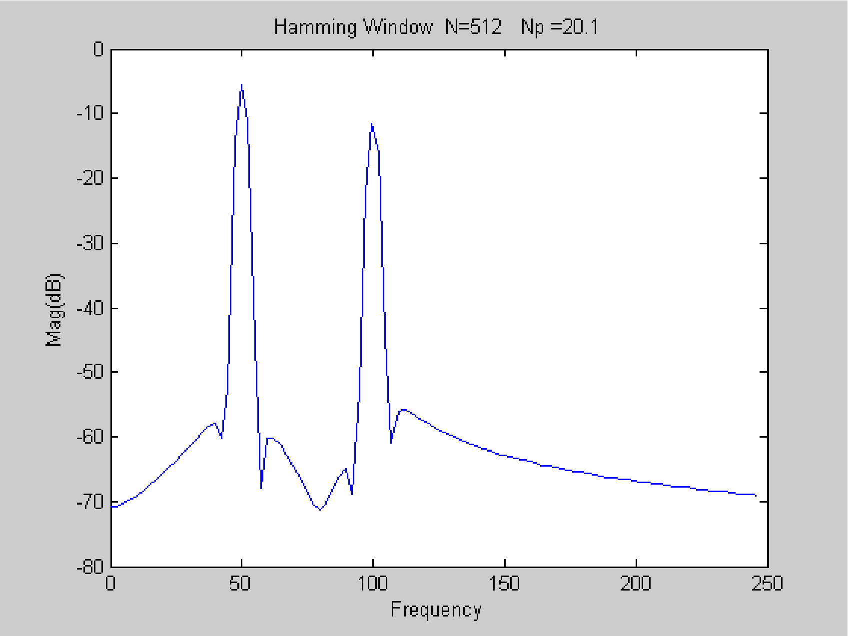


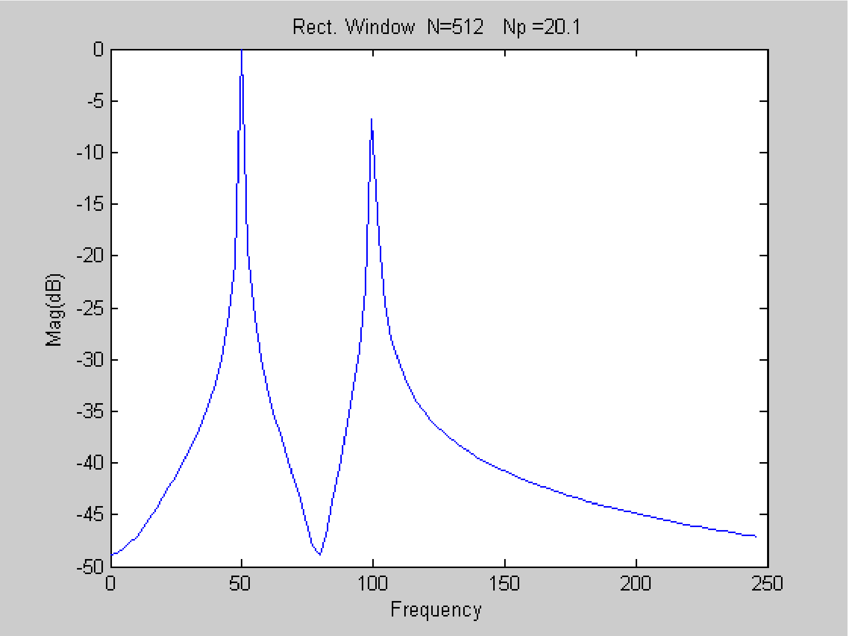
Spectral Response with Non-Coherent Sampling and Windowing



(zoomed in around fundamental)

Comparison with Rectangular Window





Hamming Window

Columns 1 through 7

-70.8278 -70.6955 -70.3703 -69.8555 -69.1502 -68.3632 -67.5133

Columns 8 through 14

-66.5945 -65.6321 -64.6276 -63.6635 -62.6204 -61.5590 -60.4199

Columns 15 through 21

-59.3204 -58.3582 -57.8735 -60.2994 -52.6273 -14.4702 -5.4343

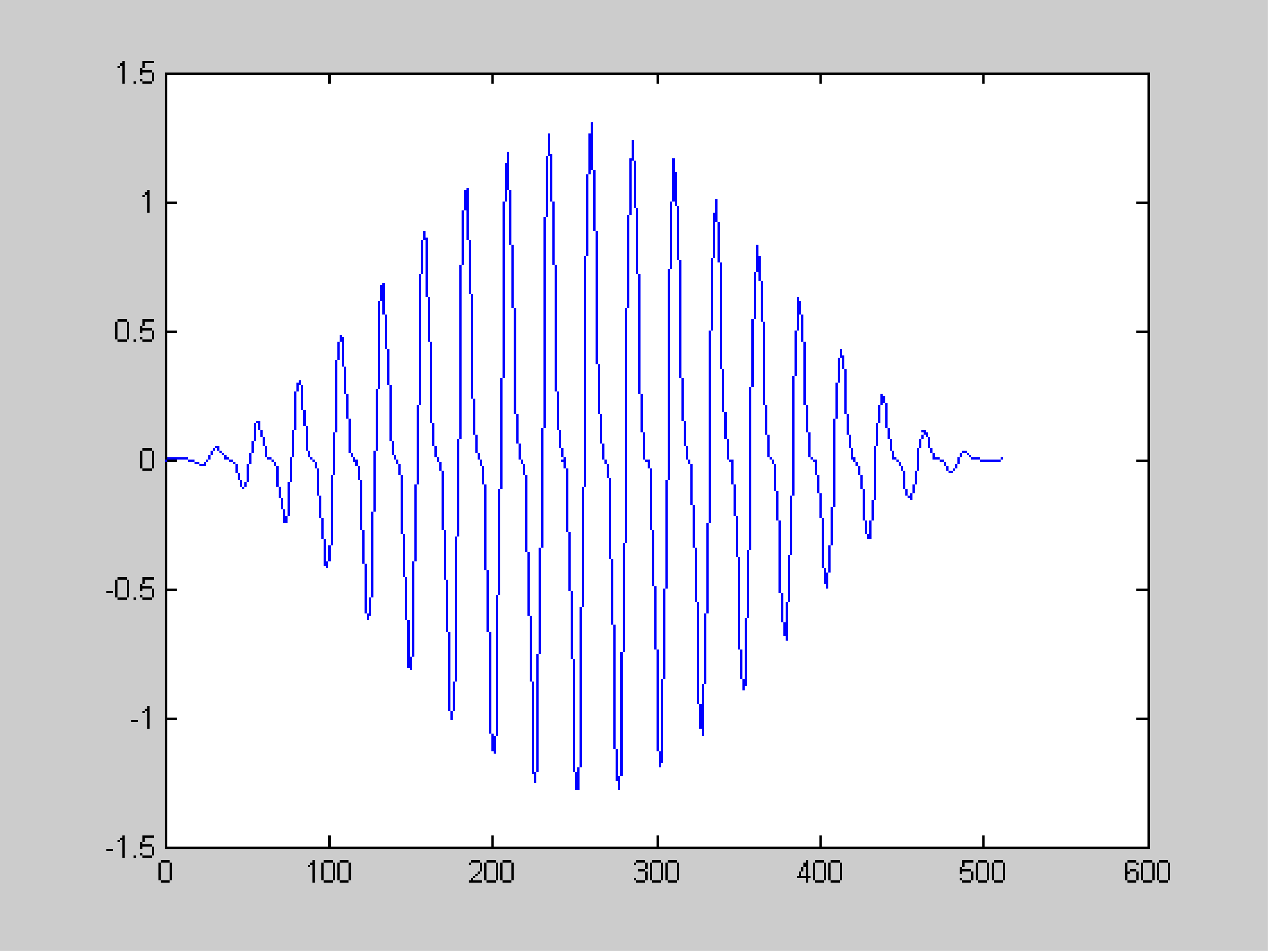
Columns 22 through 28

-11.2659 -45.2190 -67.9926 -60.1662 -60.1710 -61.2796 -62.7277

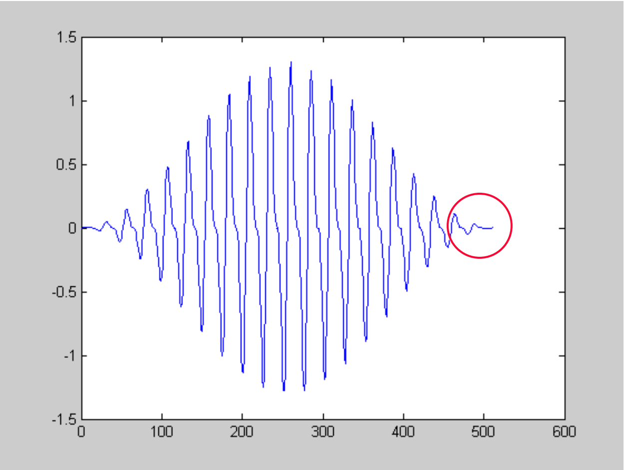
Columns 29 through 35

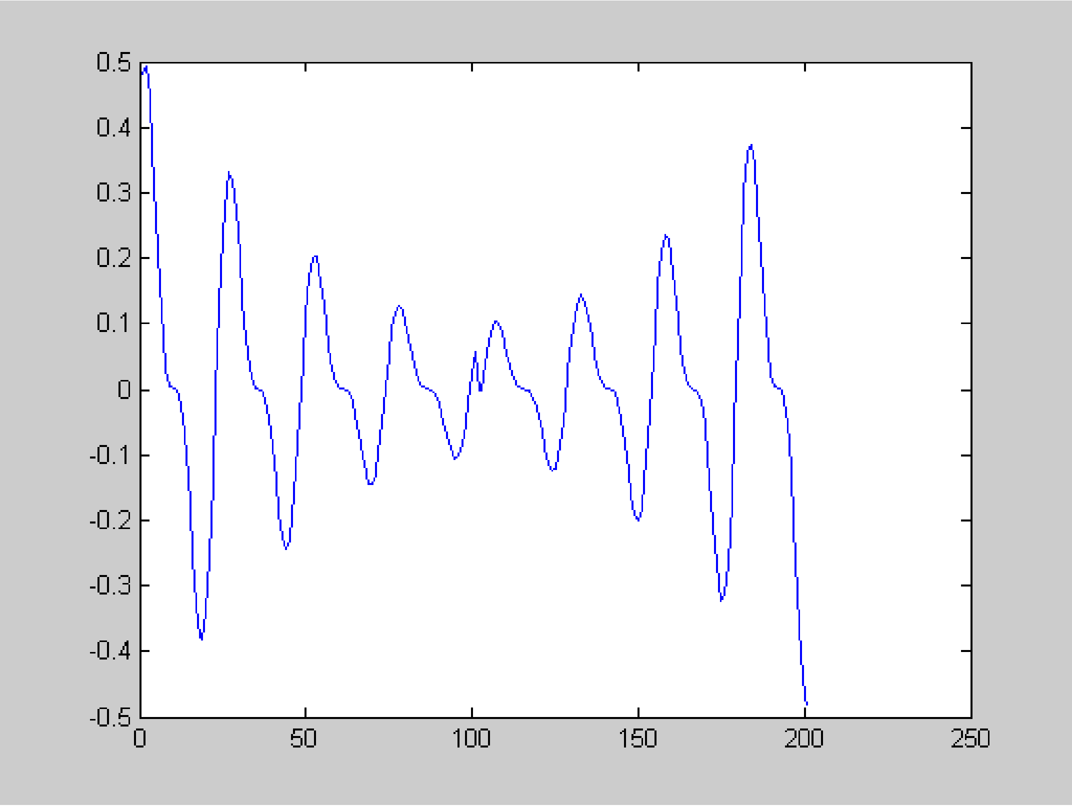
-64.3642 -66.2048 -68.2460 -70.1835 -71.1529 -70.2800 -68.1145

Hanning Window

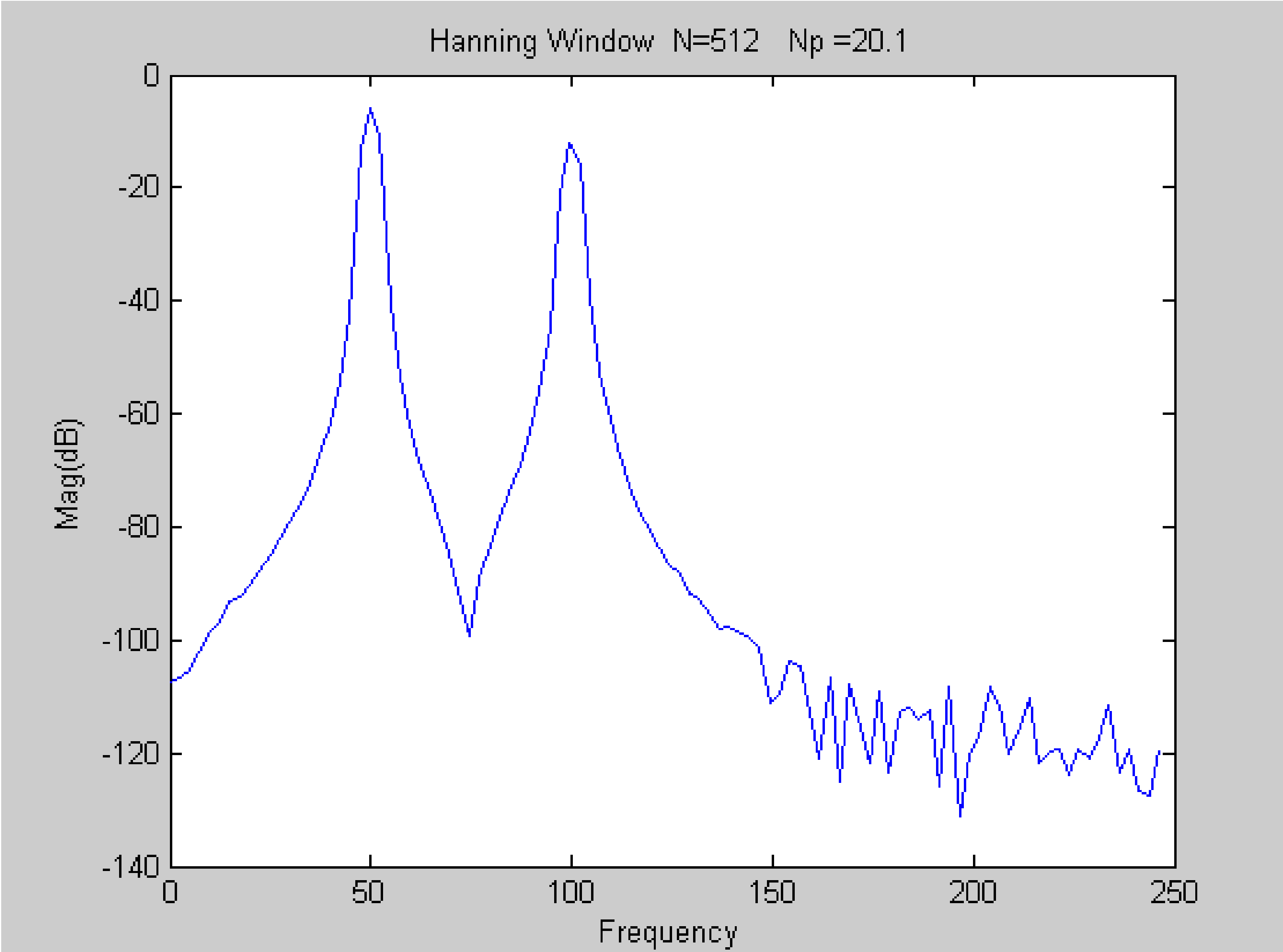


Hanning Window



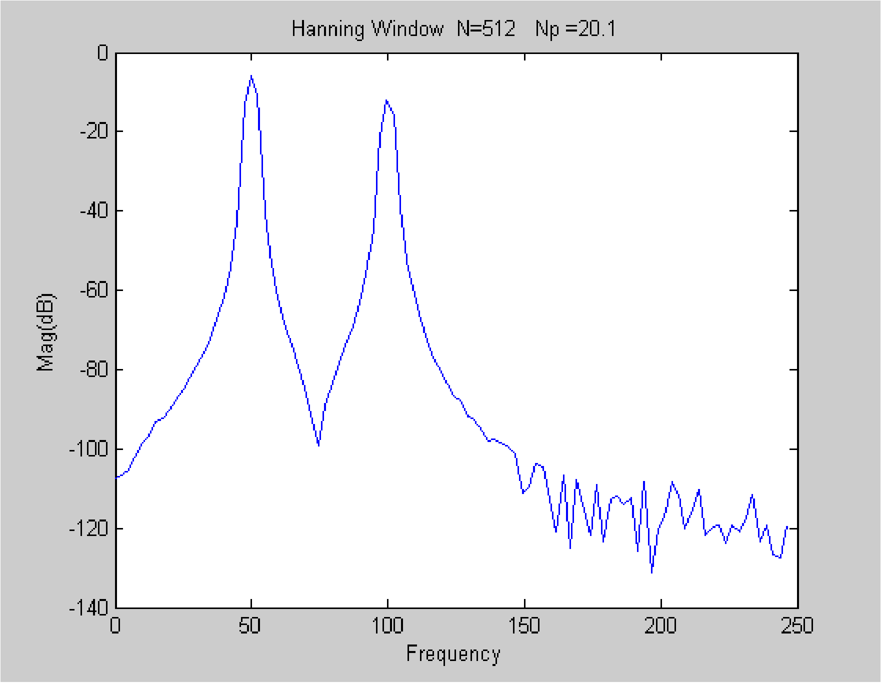


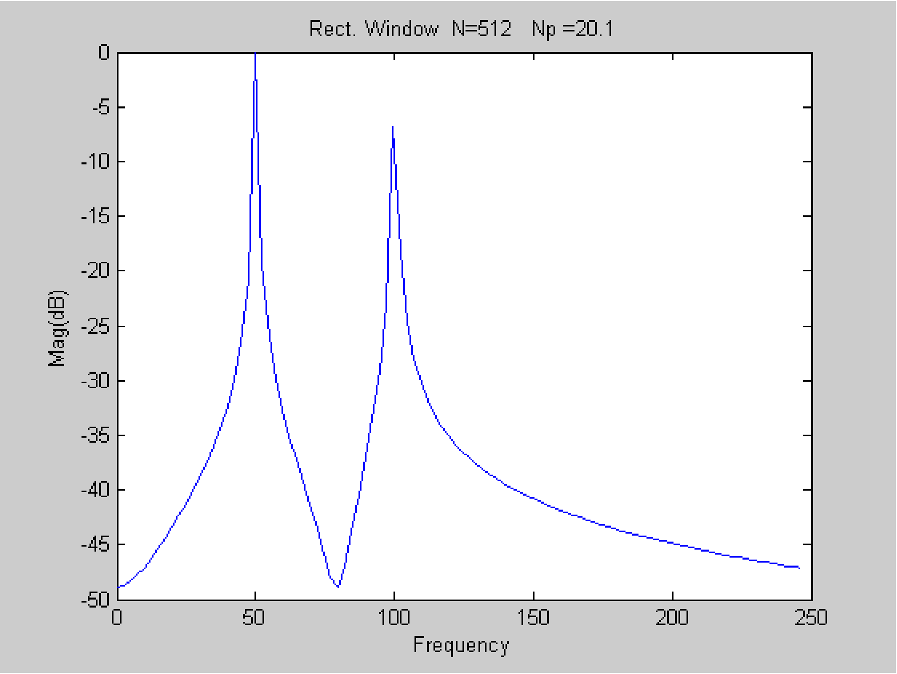
Spectral Response with Non-Coherent Sampling and Windowing



(zoomed in around fundamental)

Comparison with Rectangular Window





Hanning Window

Columns 1 through 7

-107.3123 -106.7939 -105.3421 -101.9488 -98.3043 -96.6522 -93.0343

Columns 8 through 14

-92.4519 -90.4372 -87.7977 -84.9554 -81.8956 -79.3520 -75.8944

Columns 15 through 21

-72.0479 -67.4602 -61.7543 -54.2042 -42.9597 -13.4511 -6.0601

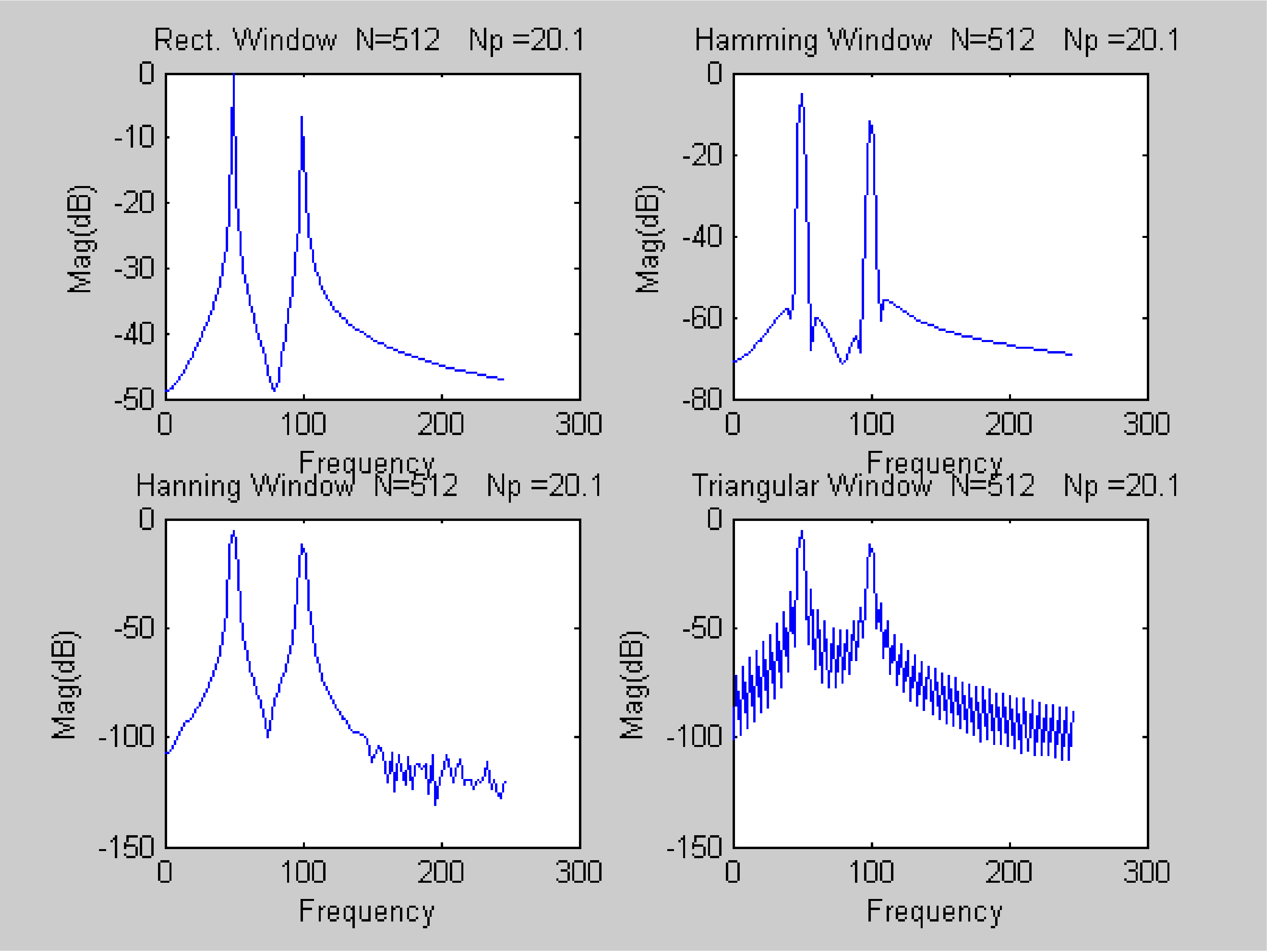
Columns 22 through 28

-10.8267 -40.4480 -53.3906 -61.8561 -68.3601 -73.9966 -79.0757

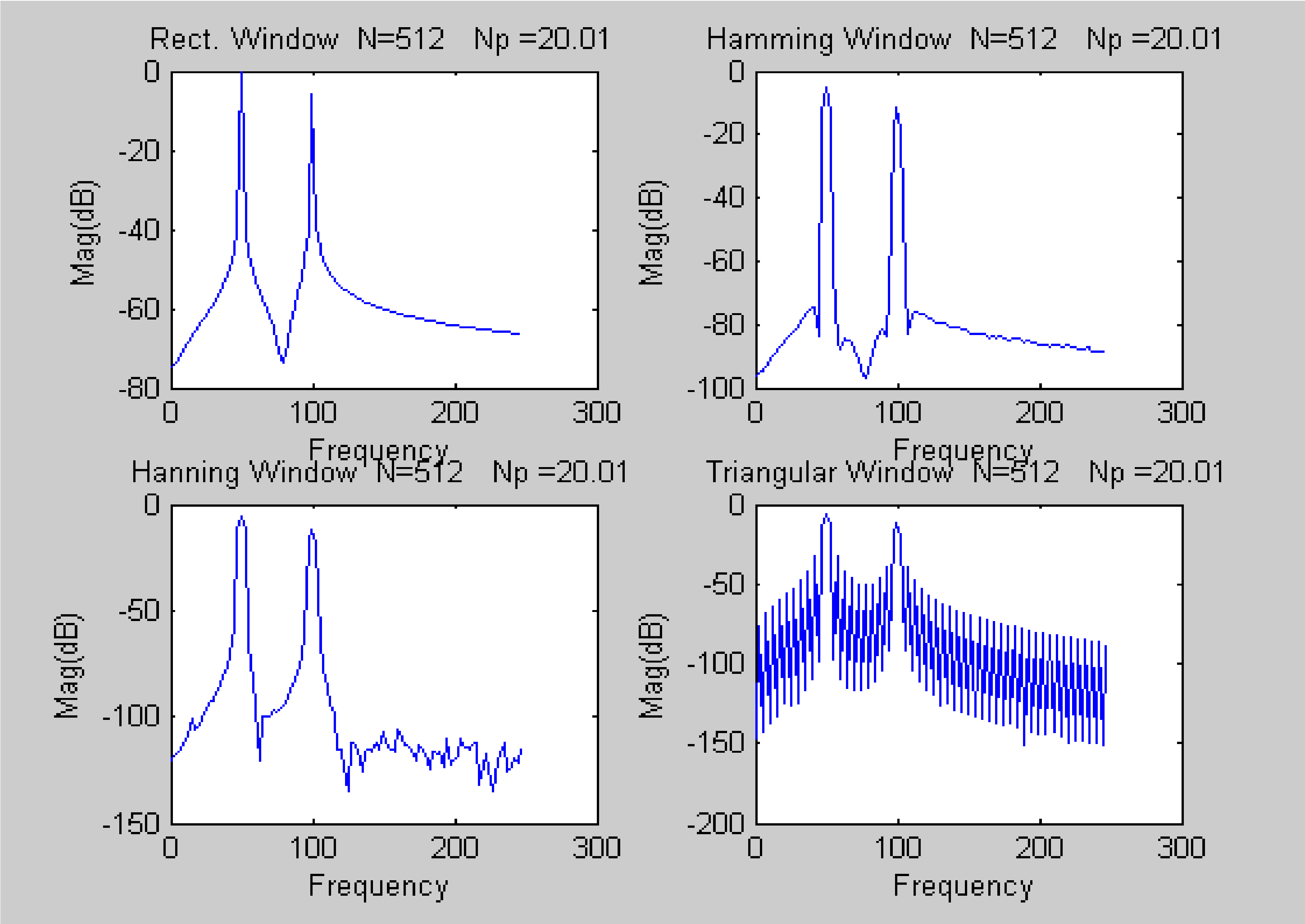
Columns 29 through 35

-84.4318 -92.7280 -99.4046 -89.0799 -83.4211 -78.5955 -73.9788

Comparison of 4 windows



Comparison of 4 windows

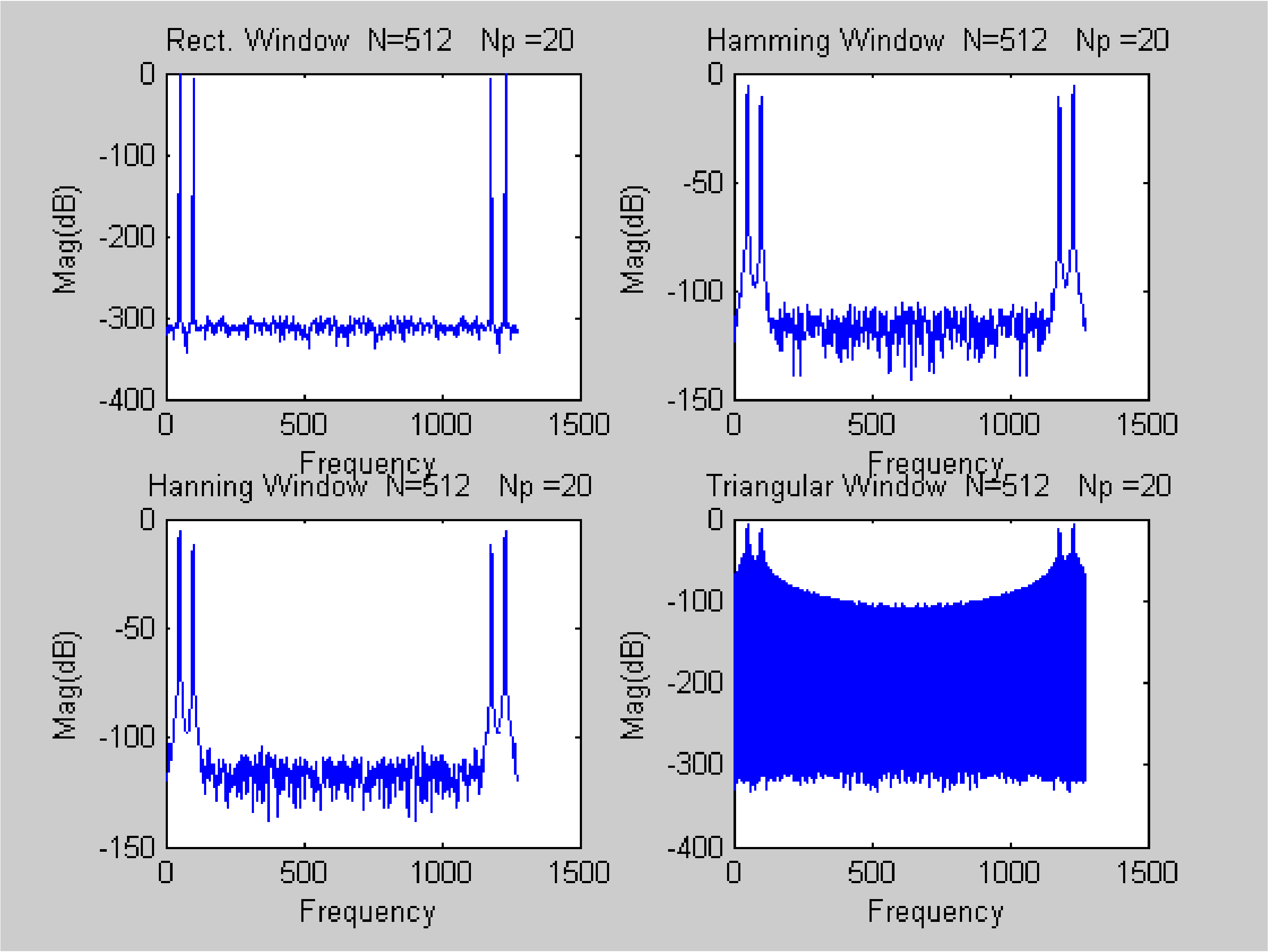


Preliminary Observations about Windows

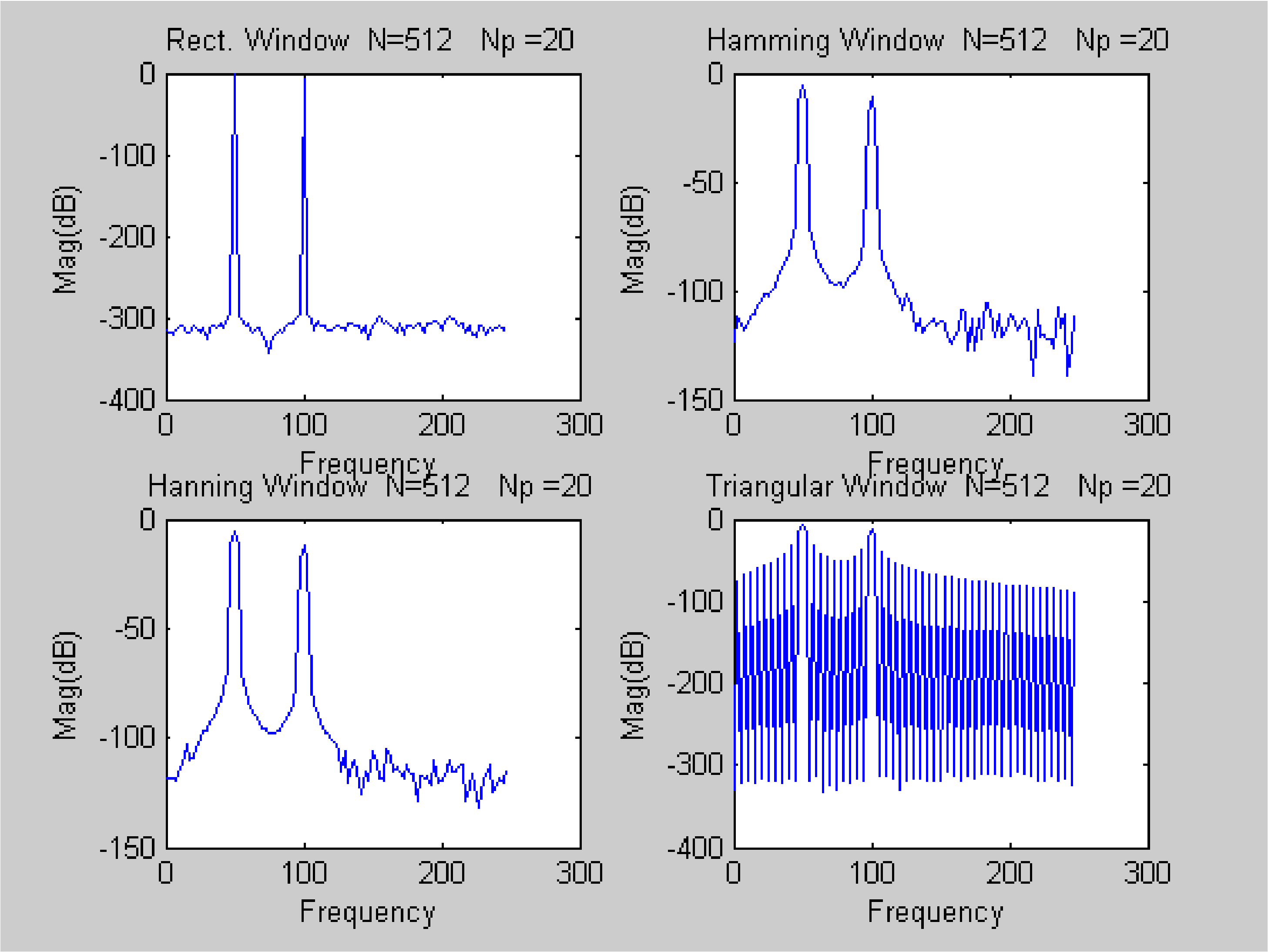
- Provide separation of spectral components
- Energy can be accumulated around spectral components
- Simple to apply
- Some windows work much better than others

But – windows do not provide dramatic improvement and …

Comparison of 4 windows when sampling hypothesis are satisfied



Comparison of 4 windows



Preliminary Observations about Windows

- Provide separation of spectral components
- Energy can be accumulated around spectral components
- Simple to apply
- Some windows work much better than others

But – windows do not provide dramatic improvement and can significantly degrade performance if sampling hypothesis are met

Addressing Spectral Analysis Challenges

- Problem Awareness
- Windowing and Filtering
- Post-processing

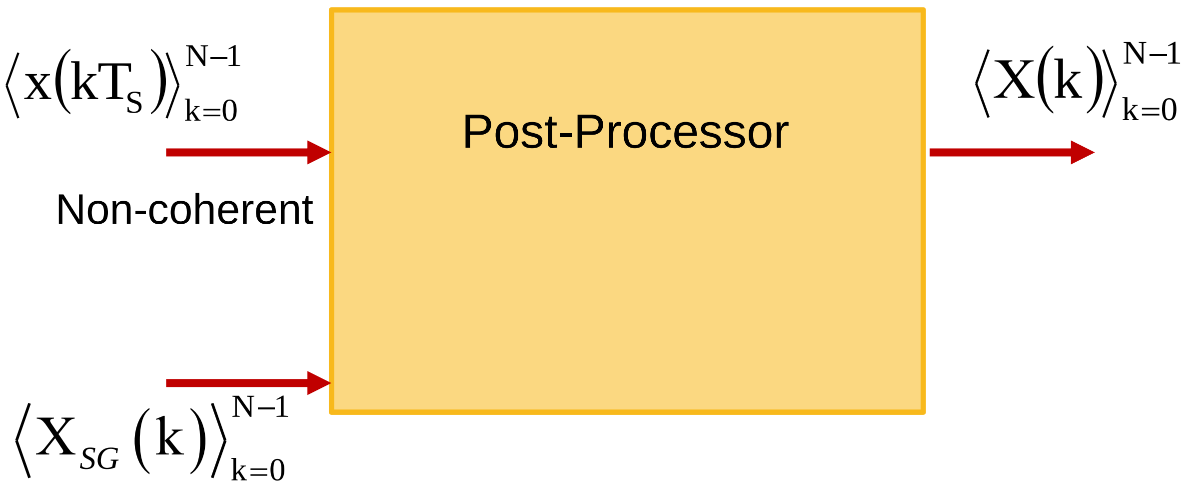
Post-processing

Method of circumventing the coherent sampling problem

Can also be used for addressing spectral purity problem for test signal generation

- Easily implemented in MATLAB
- Will be considered in the laboratory
- "Removes" fundamental from samples and replaces with coherent fundamental before taking DFT

Post-processing



- Easily implemented in MATLAB
- Will be considered in the laboratory
- “Removes” fundamental from samples and replaces with coherent fundamental before taking DFT
- Removes spectral impurity of input test signal generator when testing data converters

Issues of Concern for Spectral Analysis

An integral number of periods is critical for spectral analysis

Not easy to satisfy this requirement in the laboratory

Windowing can help but can hurt as well

Out of band energy can be reflected back into bands of interest

Characterization of CAD tool environment is essential

Spectral Characterization of high-resolution data converters requires particularly critical consideration to avoid simulations or measurements from masking real performance

Spectral Characterization of Data Converters

- Distortion Analysis
- Time Quantization Effects
  - of DACs
  - of ADCs
- Amplitude Quantization Effects
  - of DACs
  - of ADCs
- Clock Jitter

Quantization Effects

time and amplitude depicted

Zero-order sample/hold on DAC or zero-order hold on ADC interpreted output

DAC

Assume DAC will be used to generate a continuous time signal

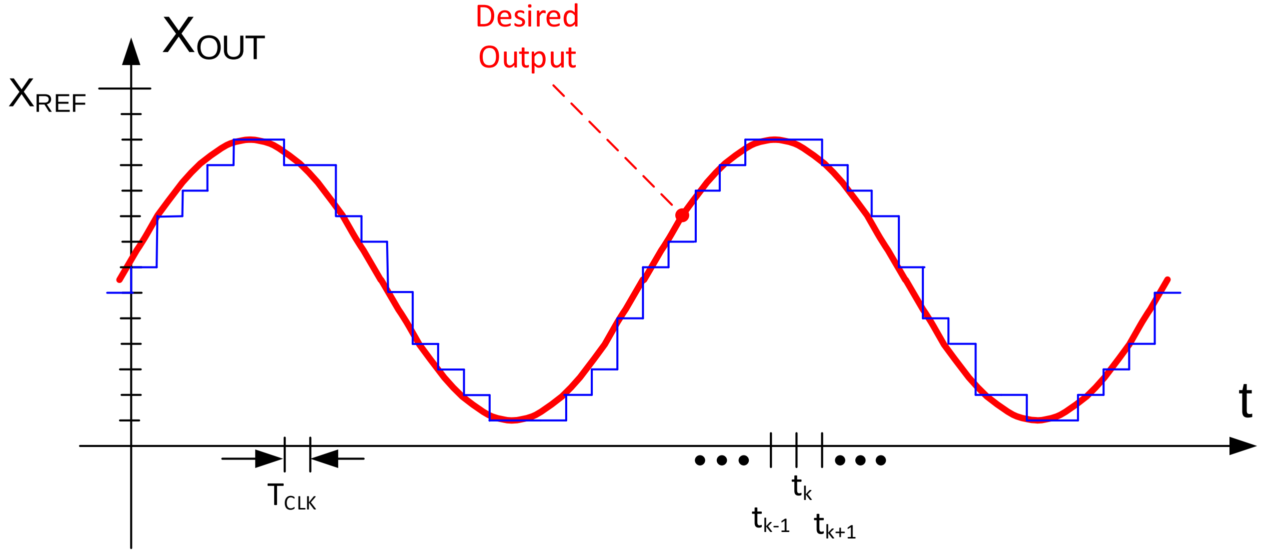
Assume DAC is driven by a clock of period T_{CLK}

DAC inputs will be a discrete sequence $\vec{X}(t_k) = \langle x_{quant}(t_k) \rangle$

DAC inputs can change only at times t_k

The duration of each DAC input depends upon system

With zero-order S/H, it is assumed that the DAC output remains constant between transaction times $x_{OUT}(t) = x_{quant}(t_k) \qquad t_k \le t < t_{k+1}$



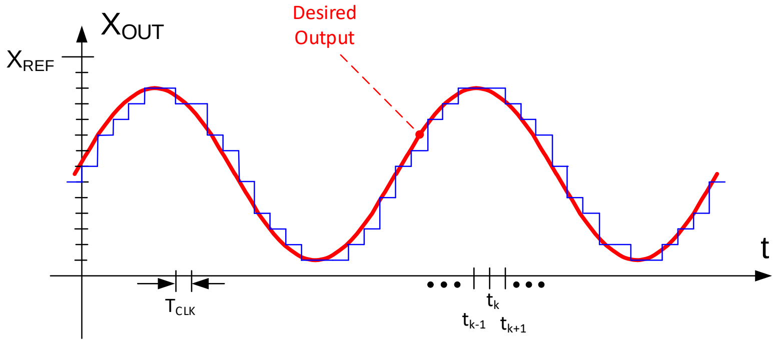
Quantization Effects

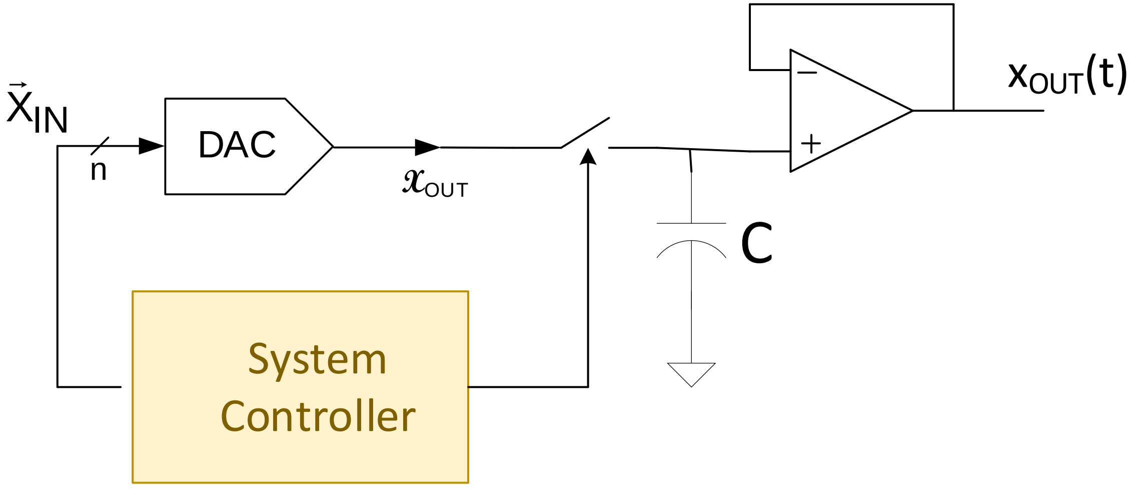
time and amplitude depicted

Zero-order sample/hold on DAC or zero-order hold on ADC interpreted output

DAC

$$x_{OUT}(t) = x_{quant}(t_k) \qquad t_k \le t < t_{k+1}$$





Zero-order S/H

Transition points not necessarily uniformly spaced but will assume so in what follows

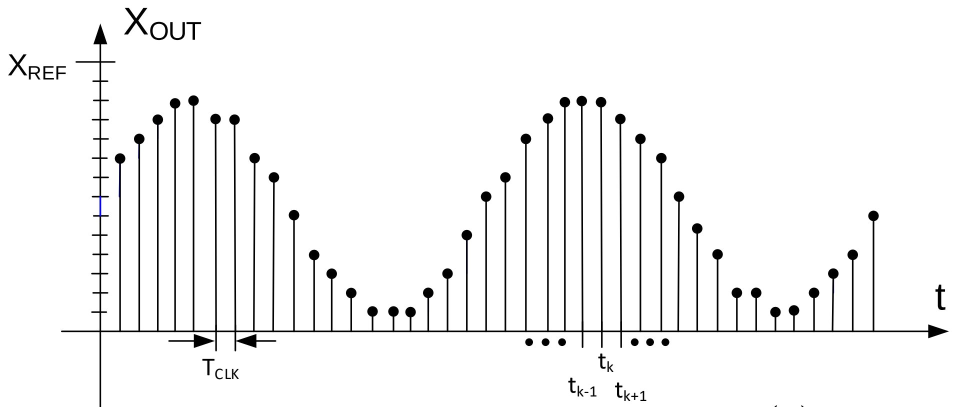
Quantization Effects

time and amplitude depicted

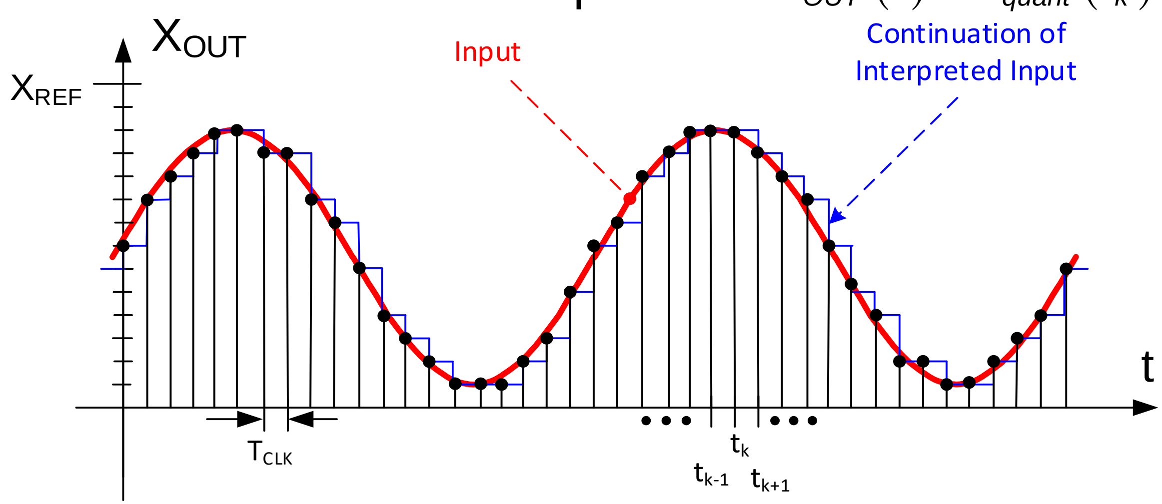
Zero-order sample/hold on DAC or zero-order hold on ADC interpreted output

ADC Output is dimensionless sequence $\vec{X}(k) = \langle x_{qant}(t_k) \rangle$

Interpreted output can be represented as a stem plot



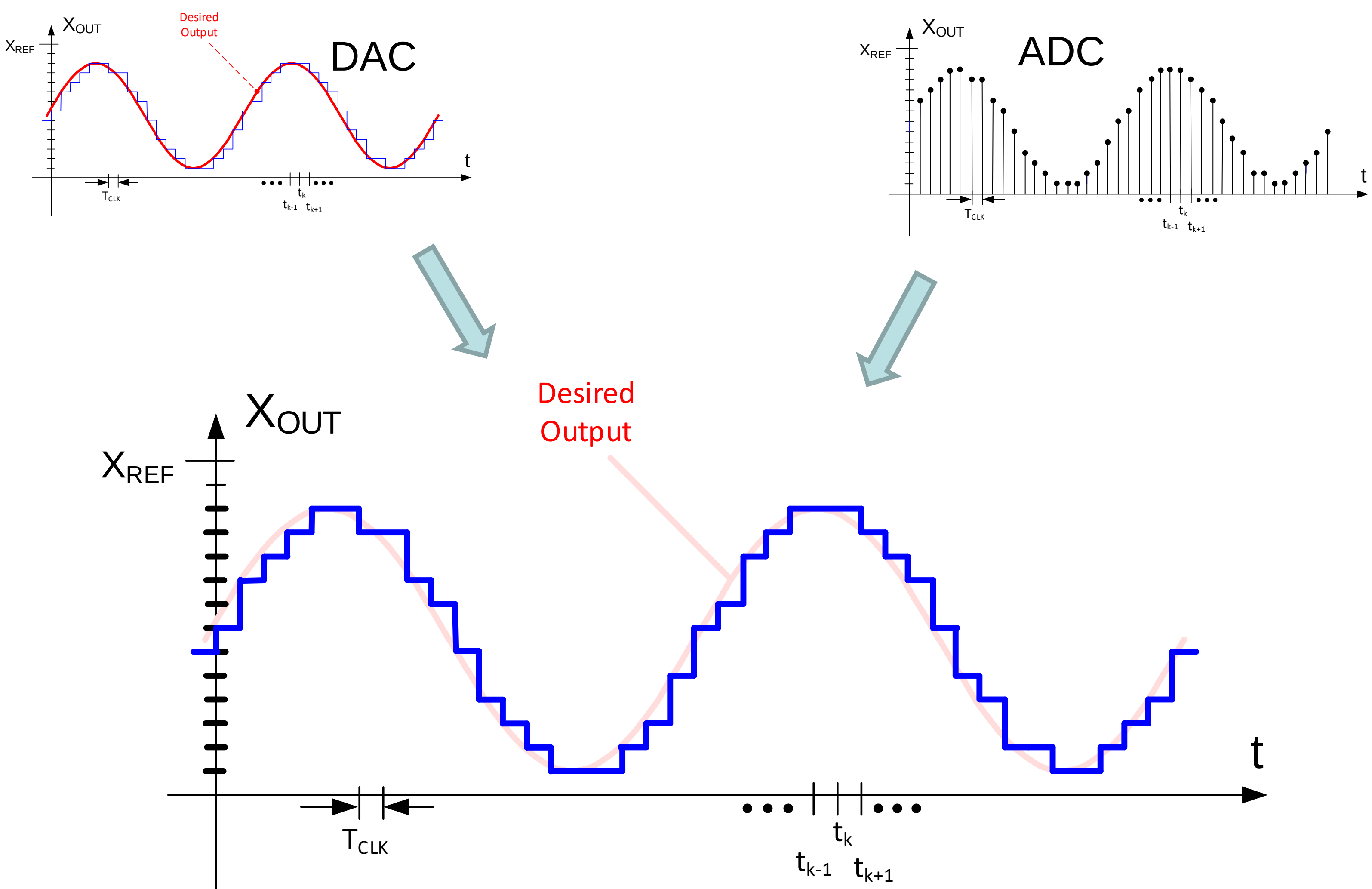
Zero-order continuation of ADC output $x_{OUT}(t) = x_{quant}(t_k) \qquad t_k \le t < t_{k+1}$



Quantization Effects

time and amplitude depicted

Zero-order sample/hold on DAC or zero-order hold on ADC interpreted output



Zero-order continuation for ADC or DAC

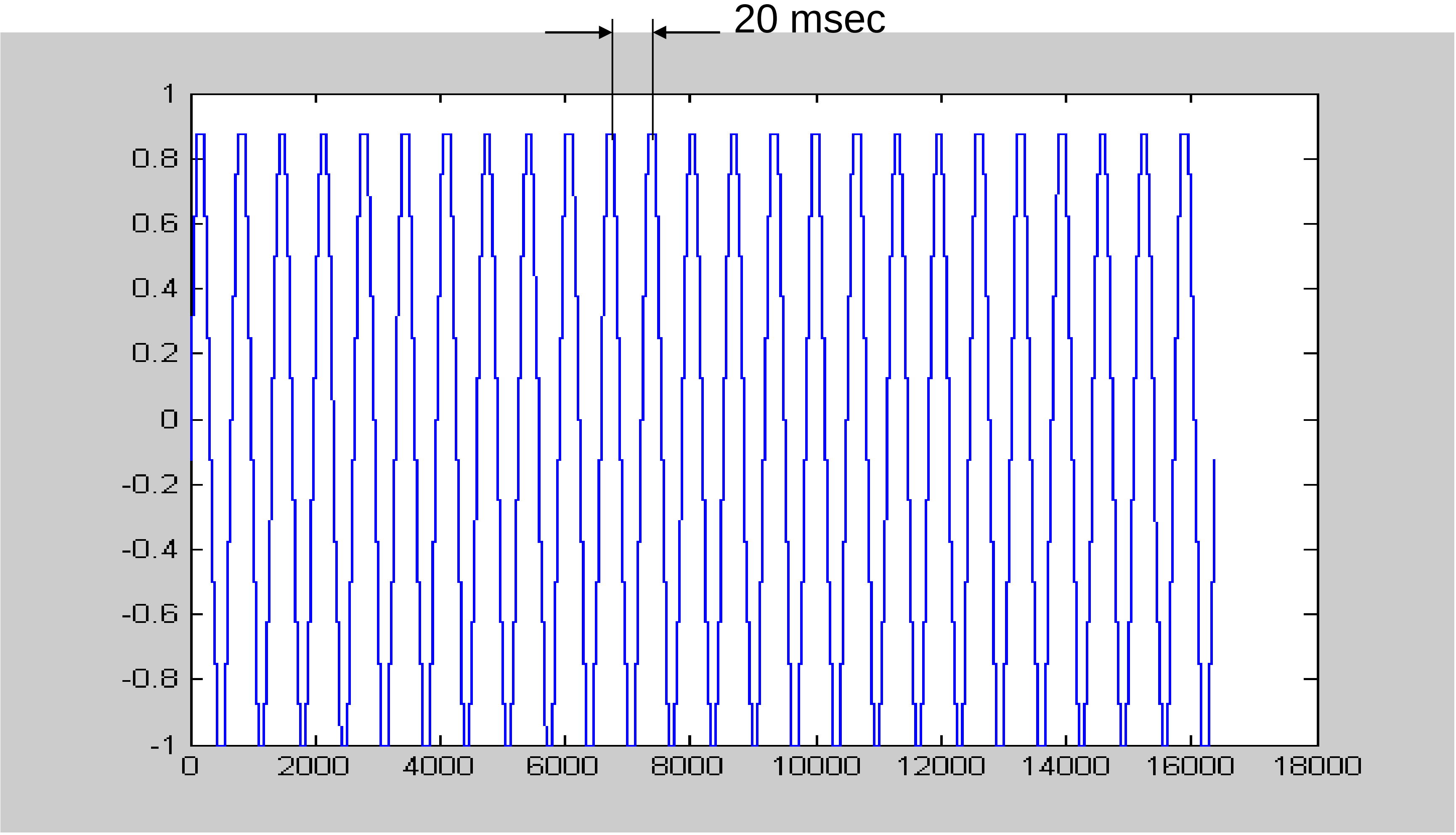
$$x_{OUT}(t) = x_{quant}(t_k) \qquad t_k \le t < t_{k+1}$$

Quantization Effects

time and amplitude depicted

For zero-order sample/hold on DAC or zero-order hold on ADC interpreted output

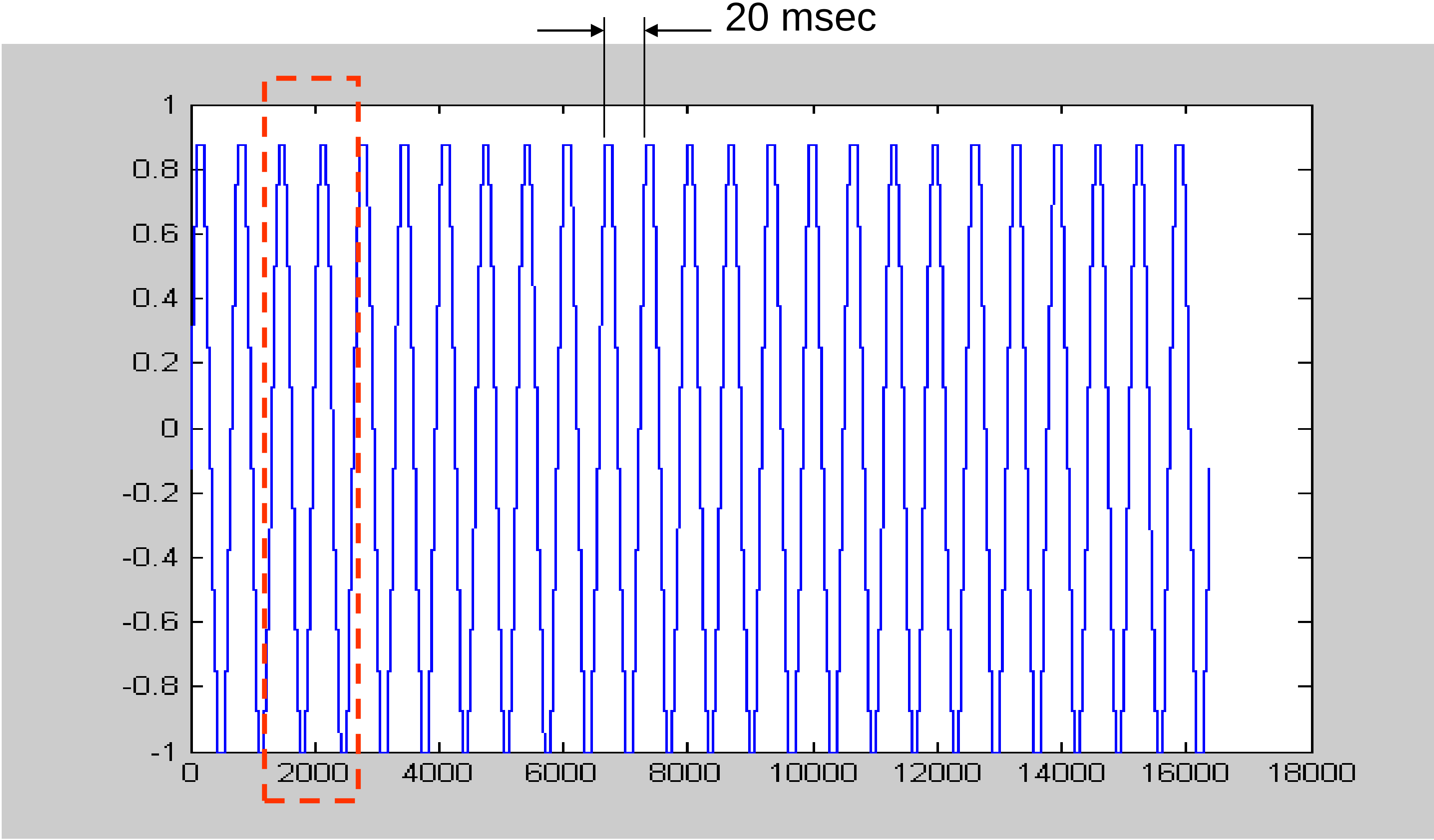
16,384 pts   res = 4bits N_P=25



Quantization Effects

(time and amplitude depicted)

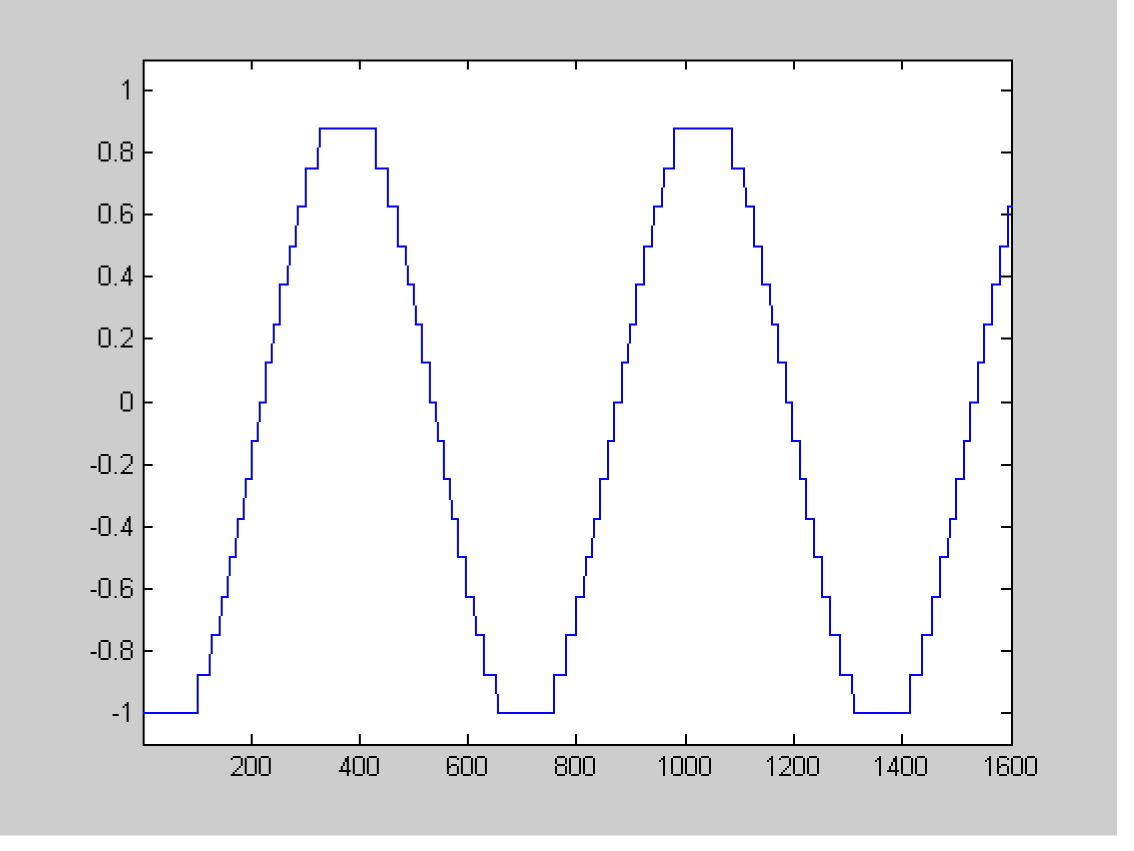
16,384 pts   res = 4bits N_P=25



Quantization Effects

(time and amplitude depicted)

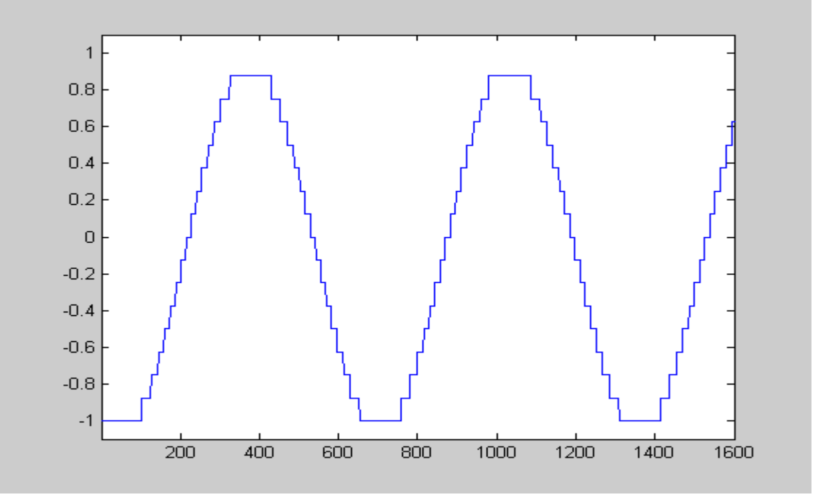
16,384 pts   res = 4bits



Is this signal band limited?

Quantization Effects

(time and amplitude depicted)



Simulation environment:

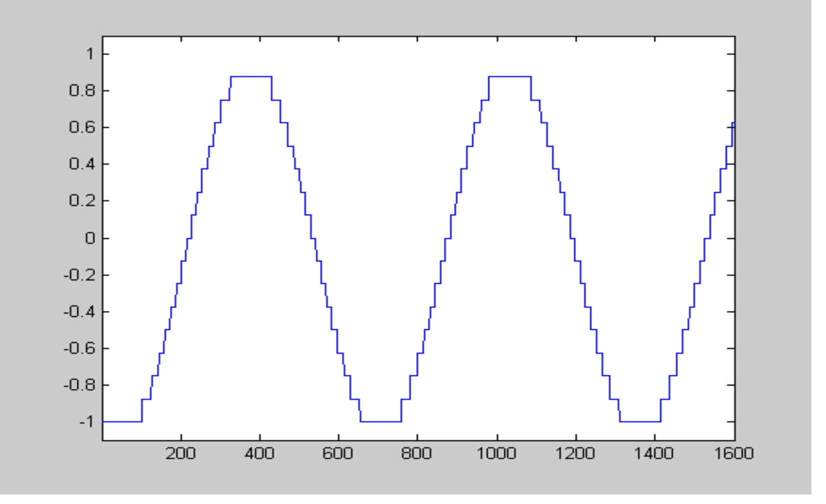
N_P=23
f_{SIG}=50Hz
V_{REF}: -1V, 1V
Res: will be varied
$N=2^n$ will be varied

Spectral Characterization of Data Converters

- Distortion Analysis
- Time Quantization Effects
  - of DACs
  - of ADCs
- Amplitude Quantization Effects
  - of DACs
  - of ADCs
- Clock Jitter

Quantization Effects

(time and amplitude depicted)



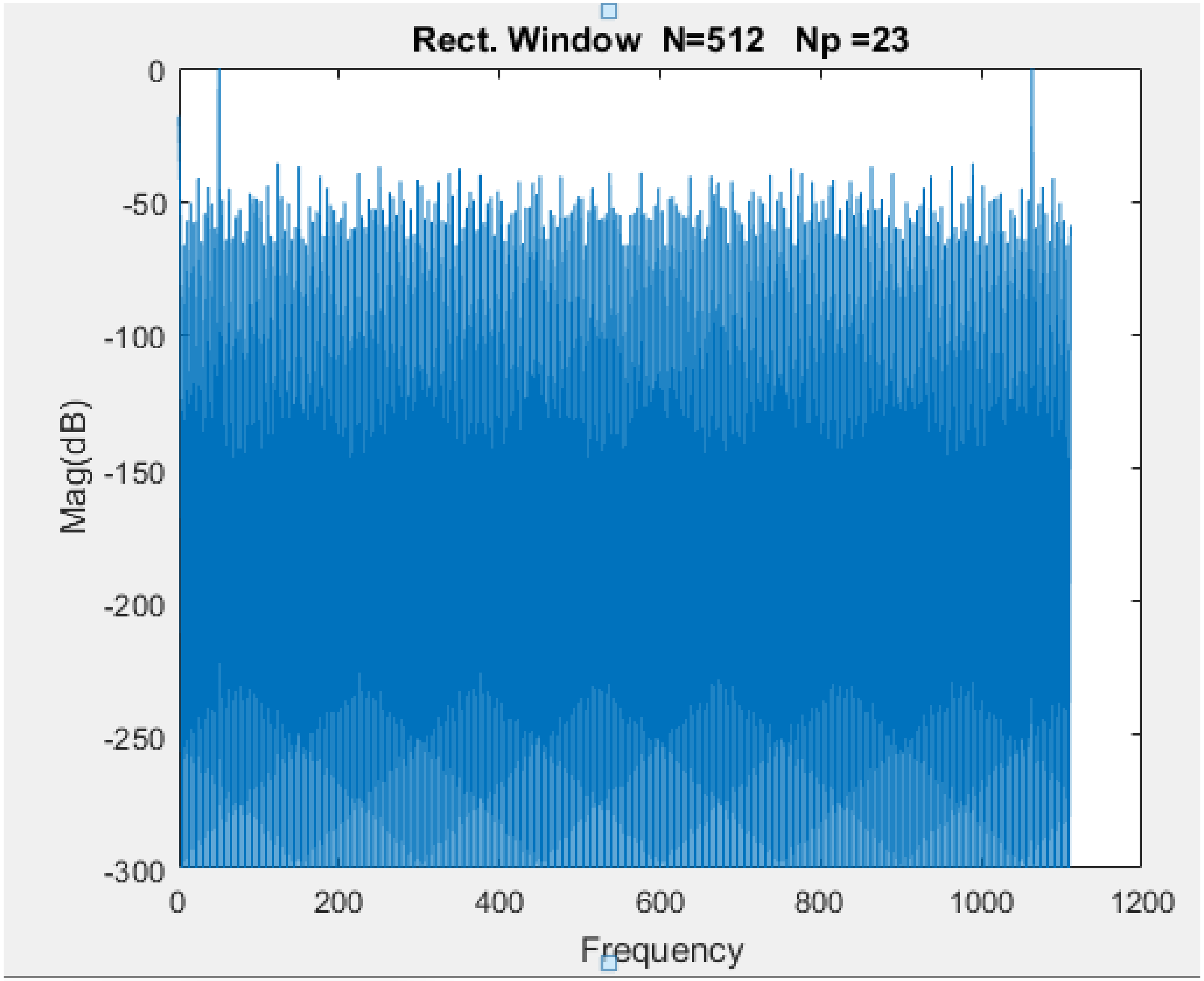
For amplitude quantization, what appear to be horizontal steps in the above figure are not the same

(amplitude quantization not depicted)

Quantization Effects

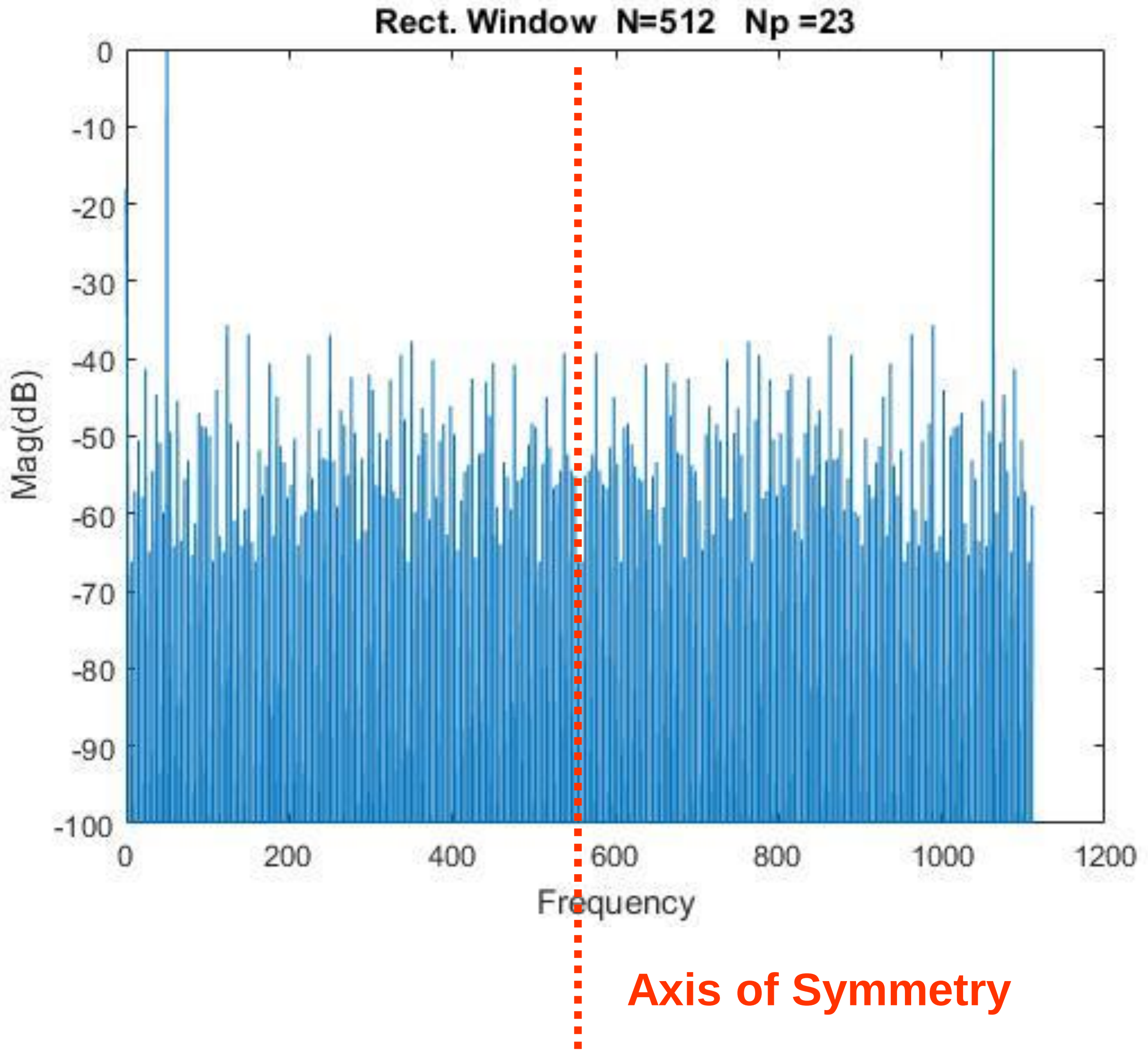
Res = 4 bits

(At each time sample, quantize the amplitude value)



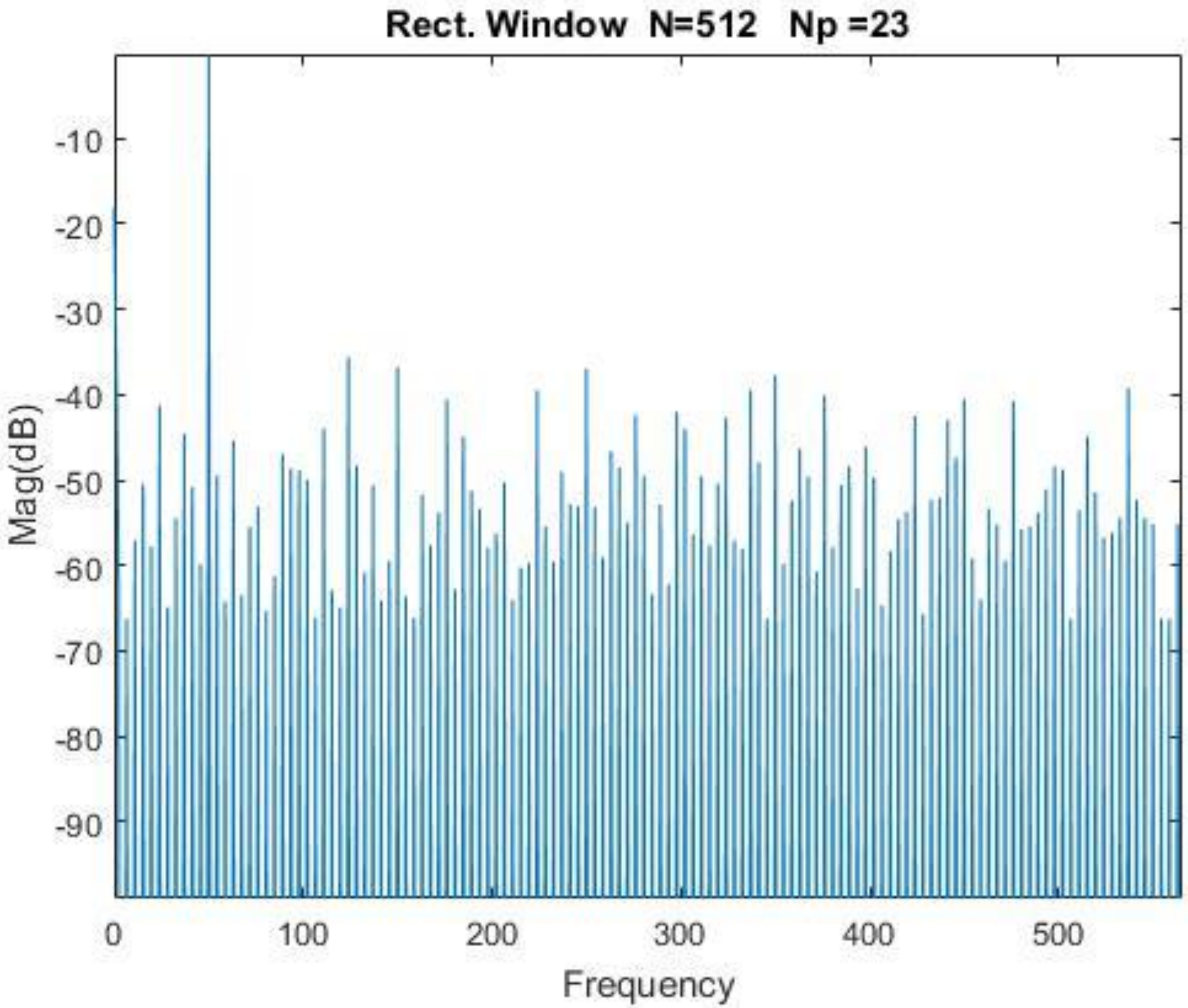
Quantization Effects

Res = 4 bits



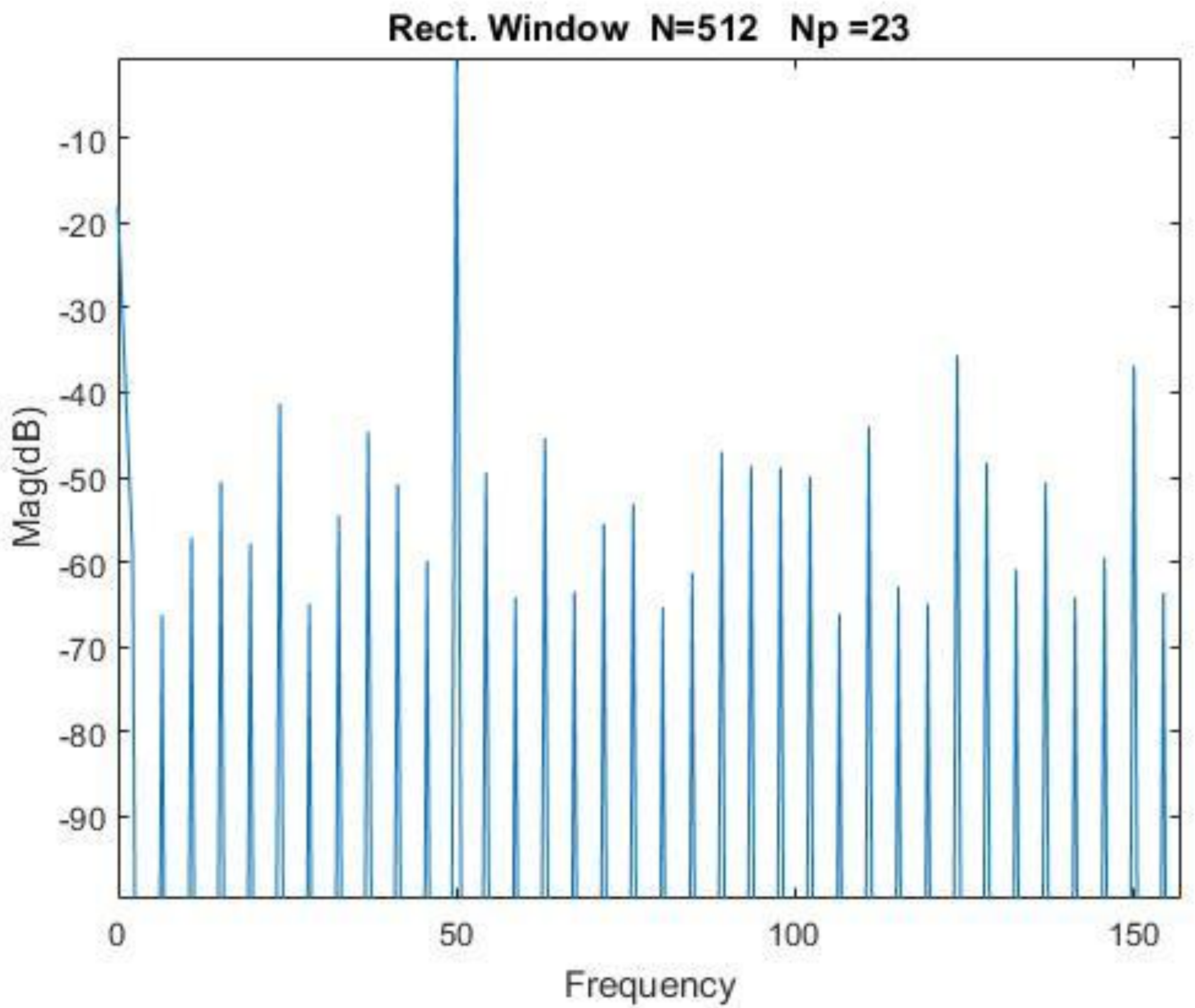
Quantization Effects

Res = 4 bits



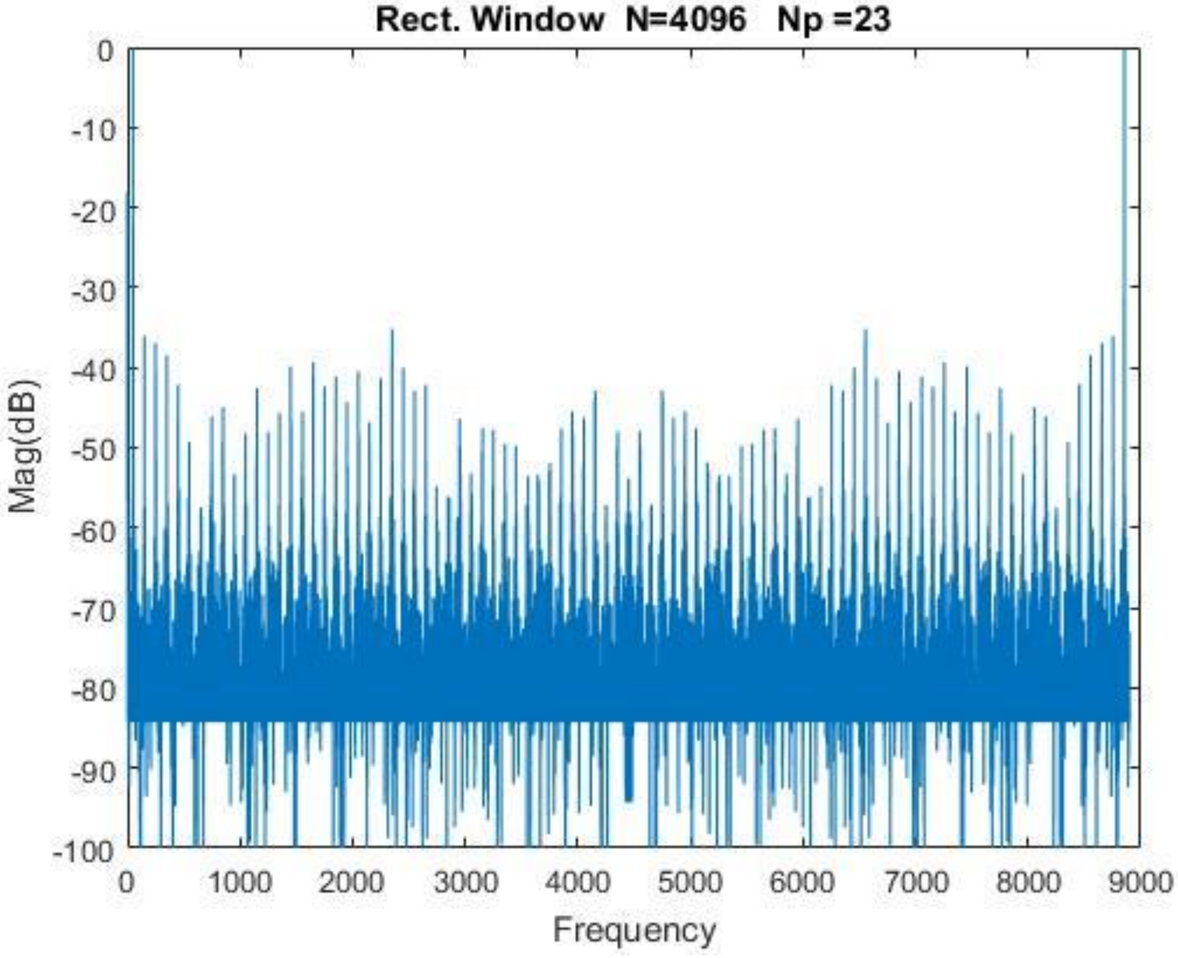
Quantization Effects

Res = 4 bits



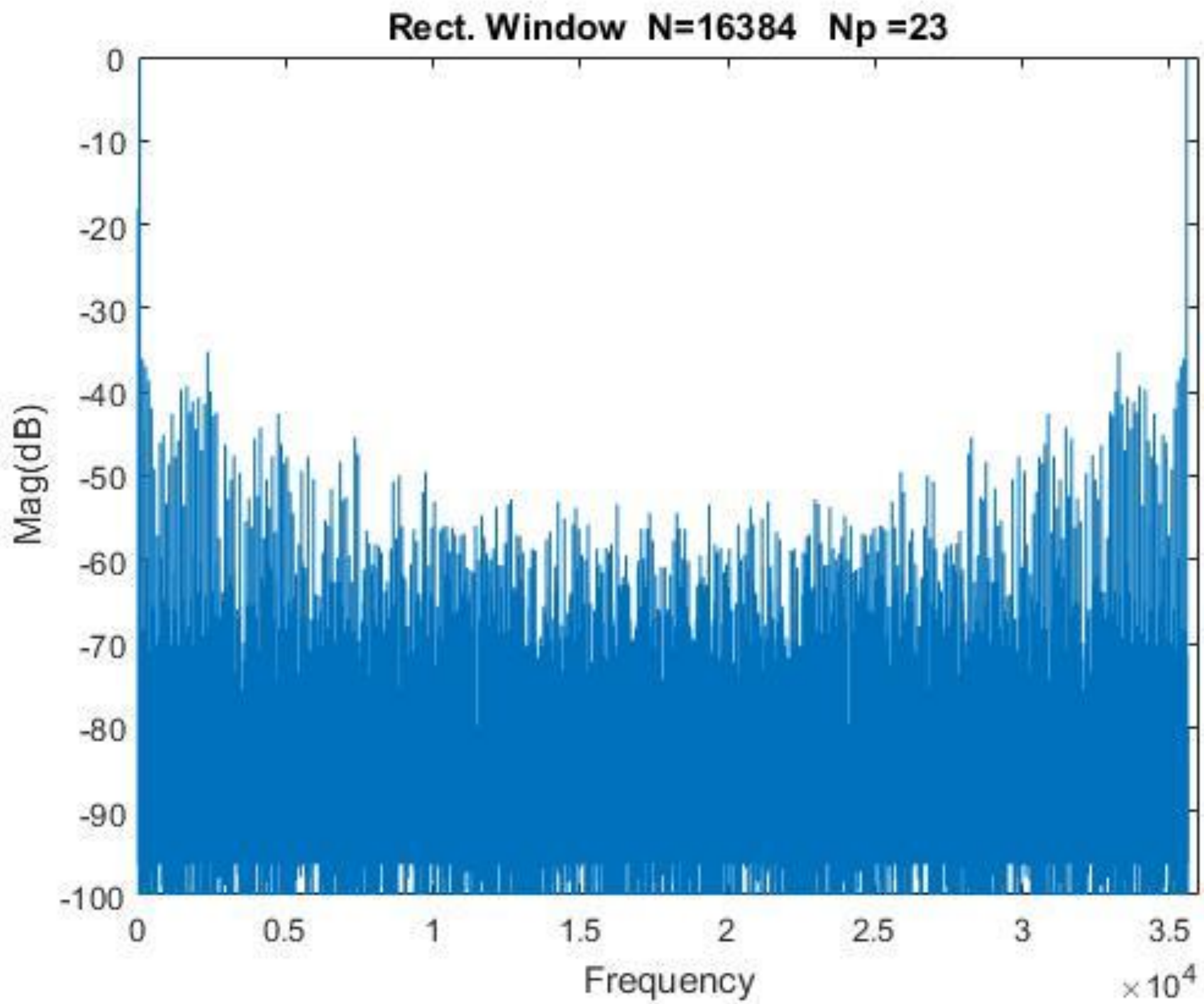
Quantization Effects

Res = 4 bits



Quantization Effects

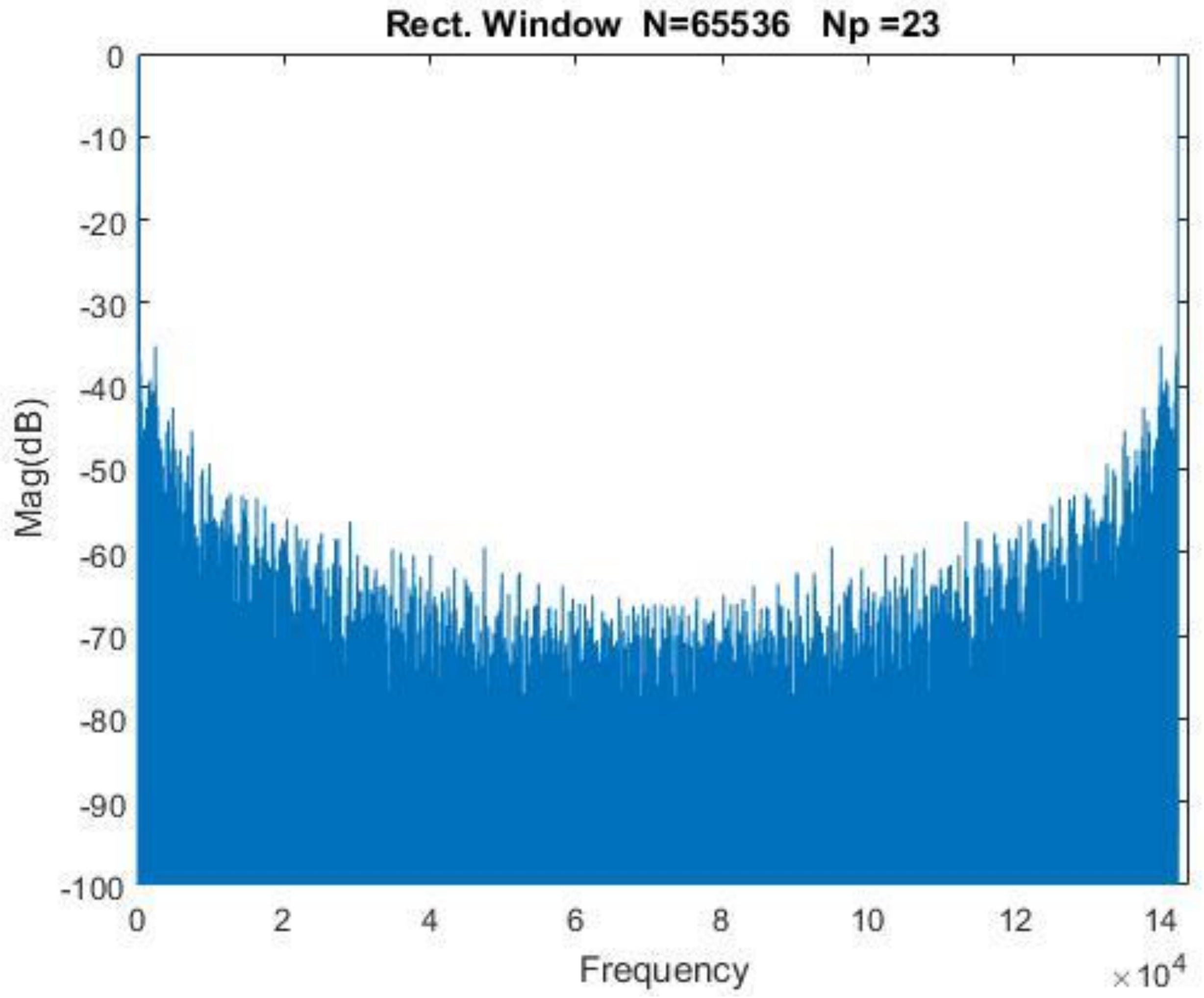
Res = 4 bits



Expect quantization noise effects to be uniformly distributed !!

Quantization Effects

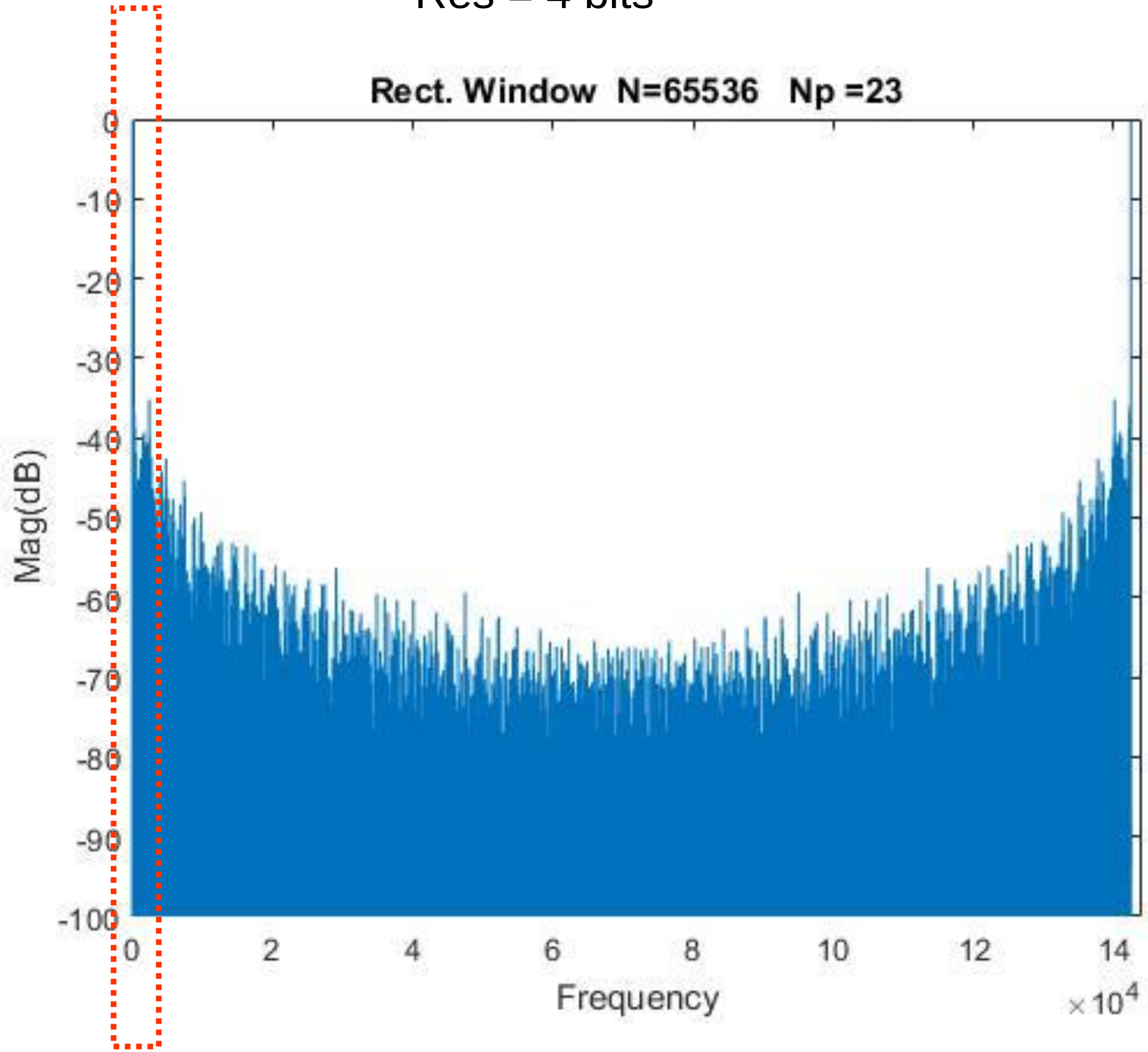
Res = 4 bits



Expect quantization noise effects to be uniformly distributed !!

Quantization Effects

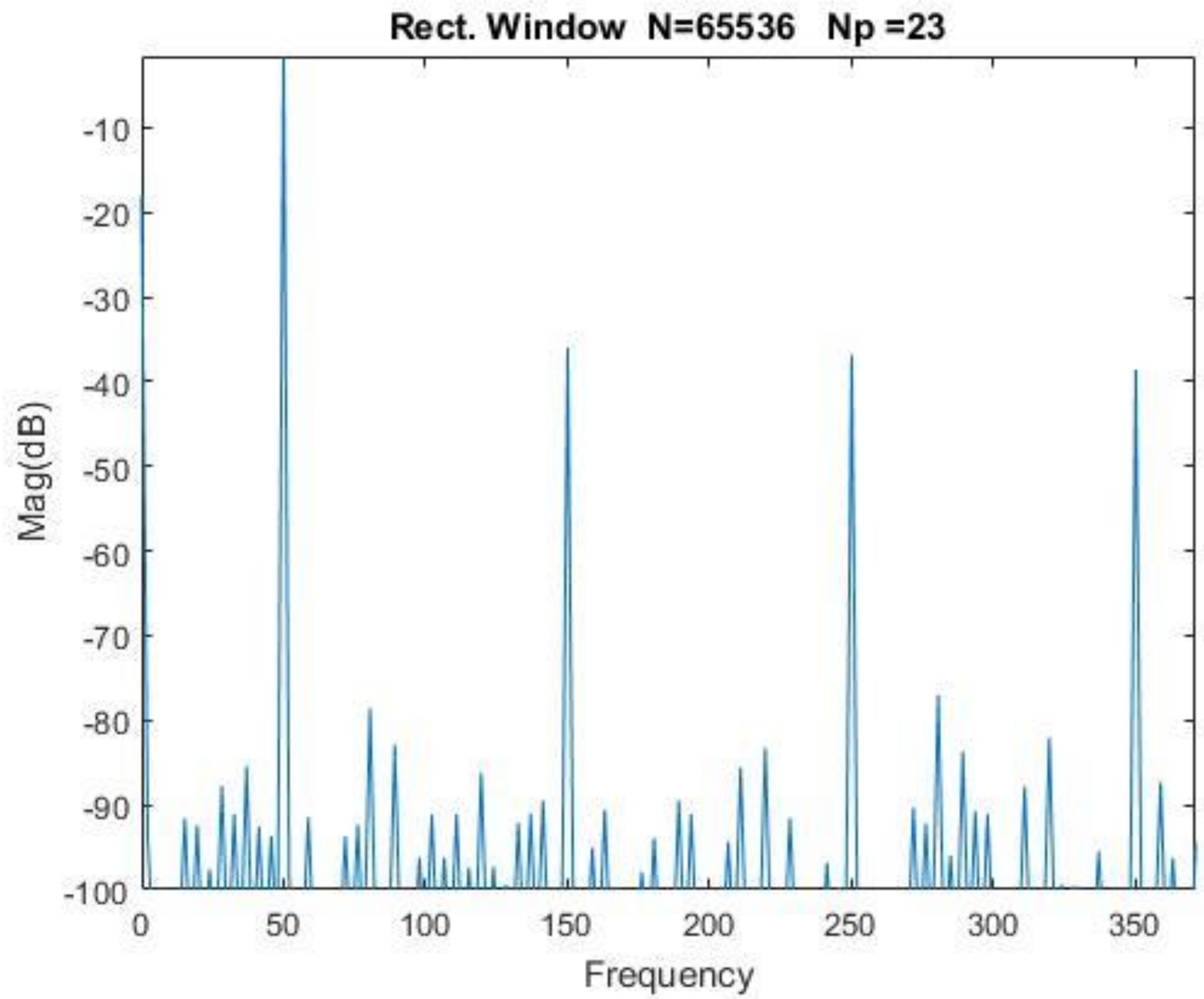
Res = 4 bits



Expect quantization noise effects to be uniformly distributed !!

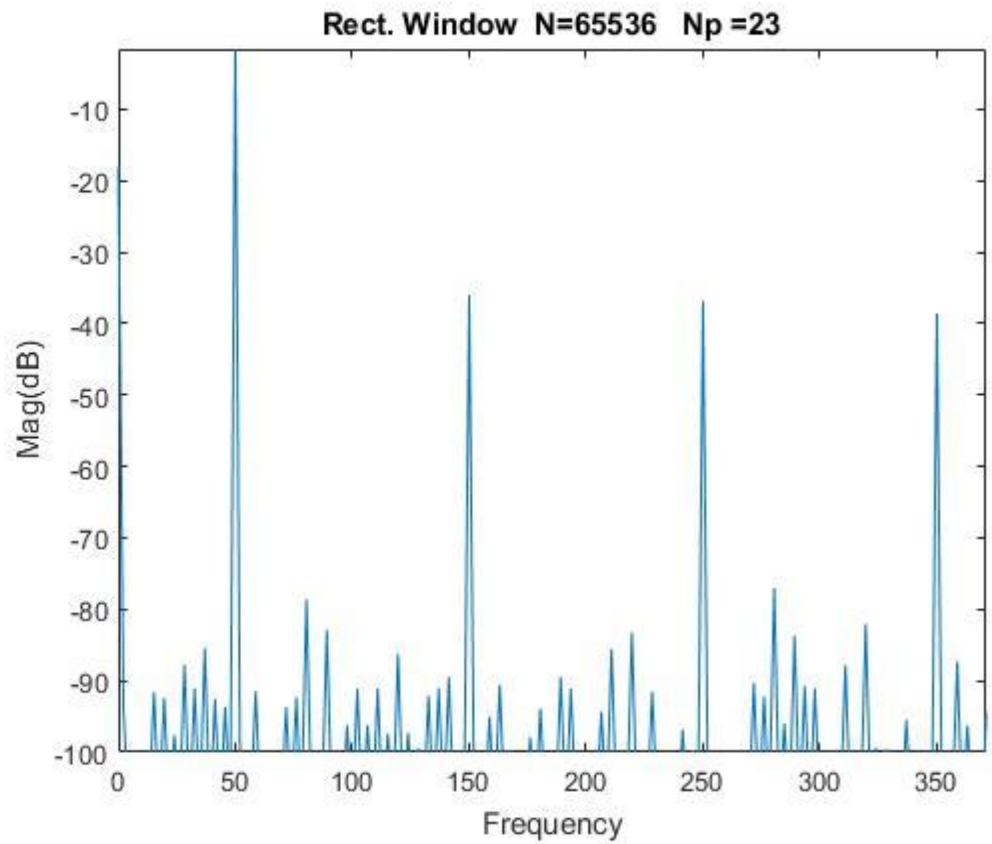
Quantization Effects

Res = 4 bits



Note presence of odd-ordered harmonic terms !!

Res = 4 bits



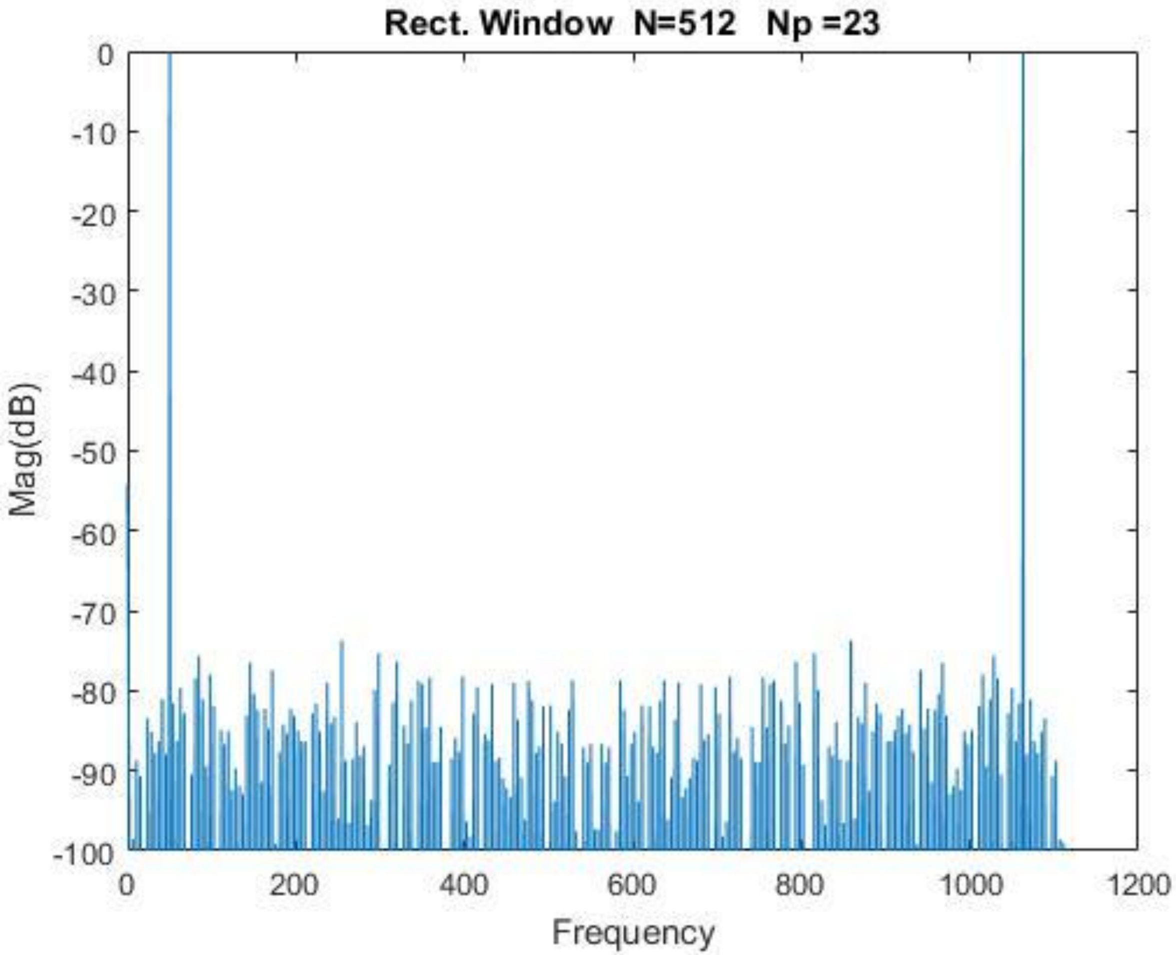
Res (n)	SNR_{corr}	SNR
1	3.86	7.78
2	12.06	13.8
3	19.0	19.82
4	25.44	25.84
5	31.66	31.86
6	37.79	37.88
8	49.90	49.92
10	61.95	61.96

Why are there spectral components present in the quantization noise?

Recall the uncorrelated assumption was good only for about 4 bits or more !

Quantization Effects

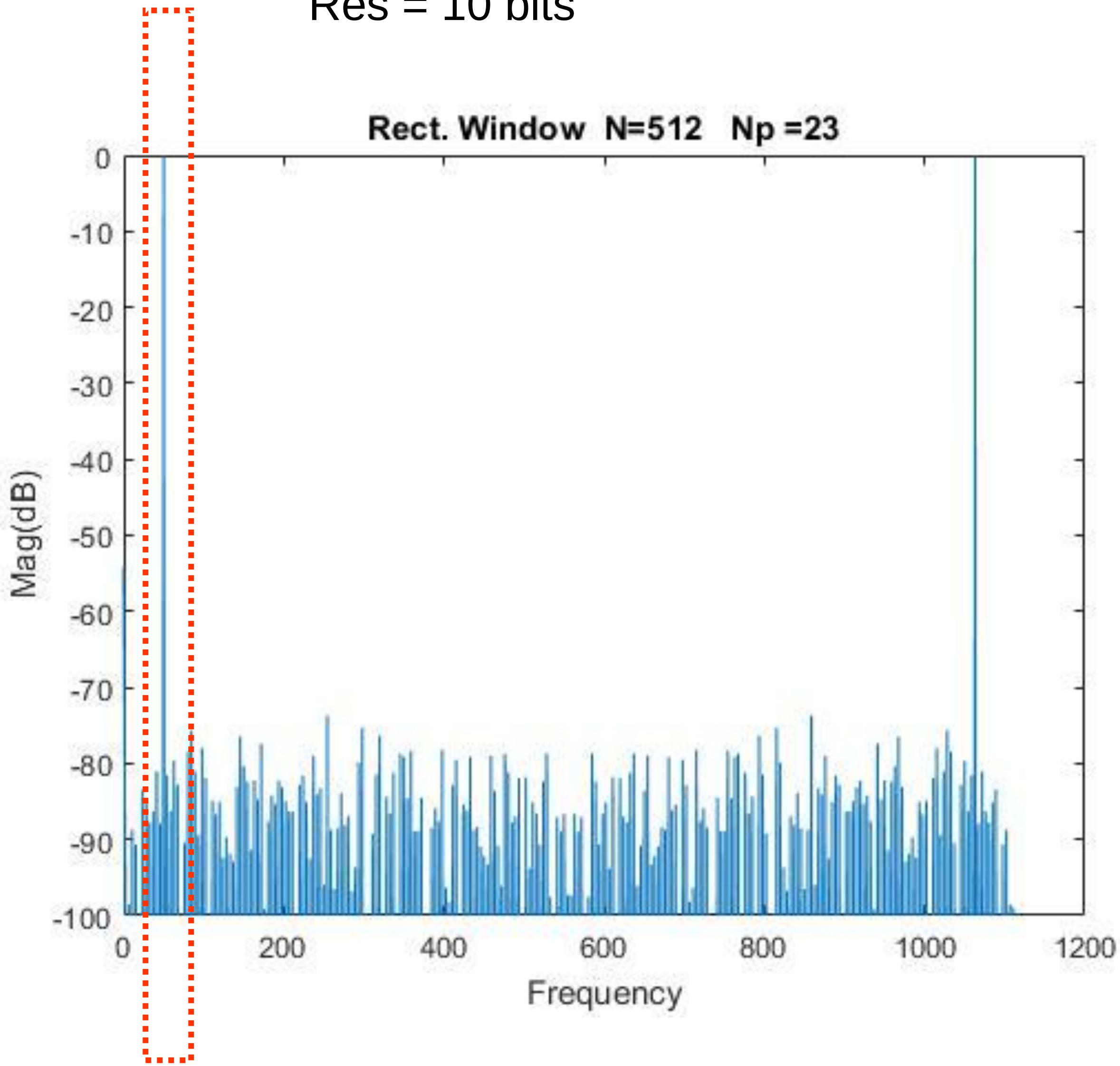
Res = 10 bits



Quantization noise is much more uniform

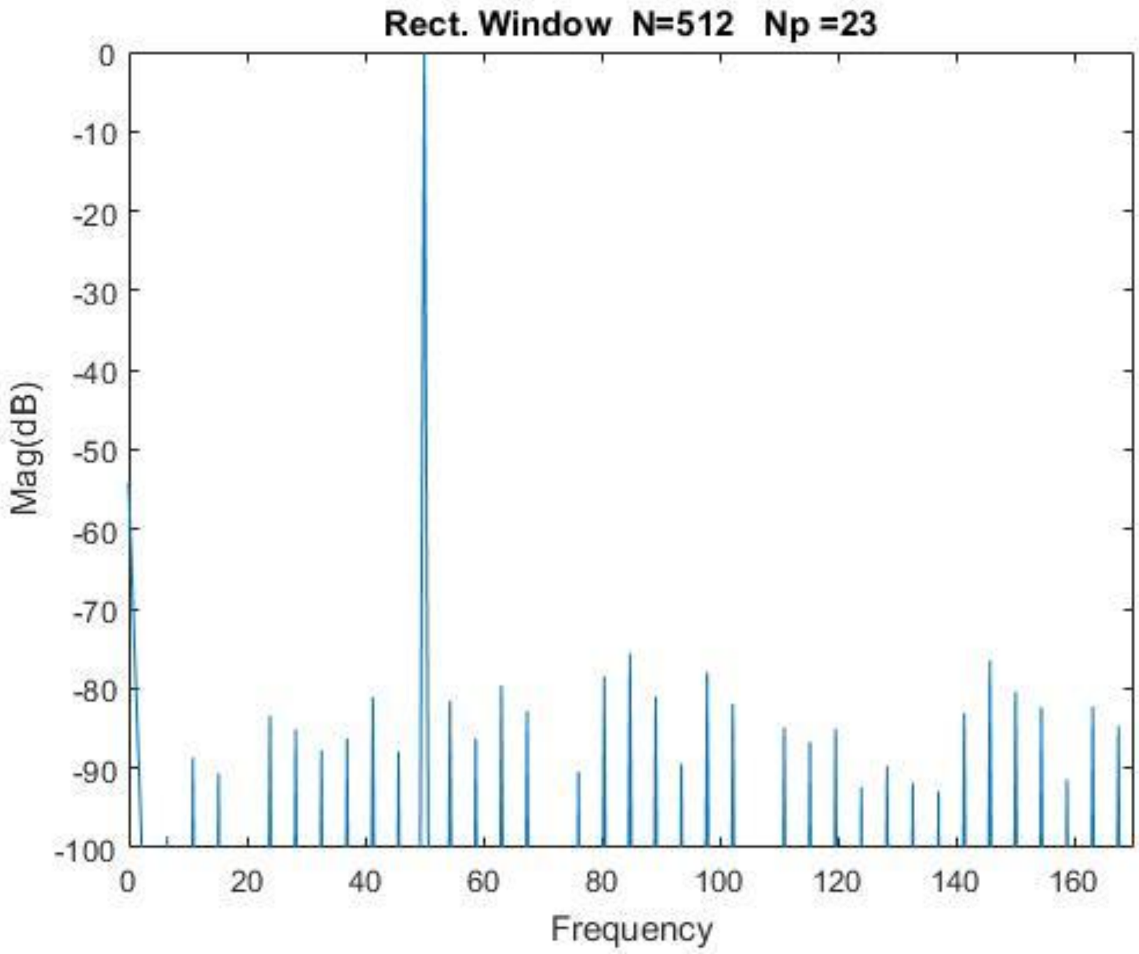
Quantization Effects

Res = 10 bits



Quantization Effects

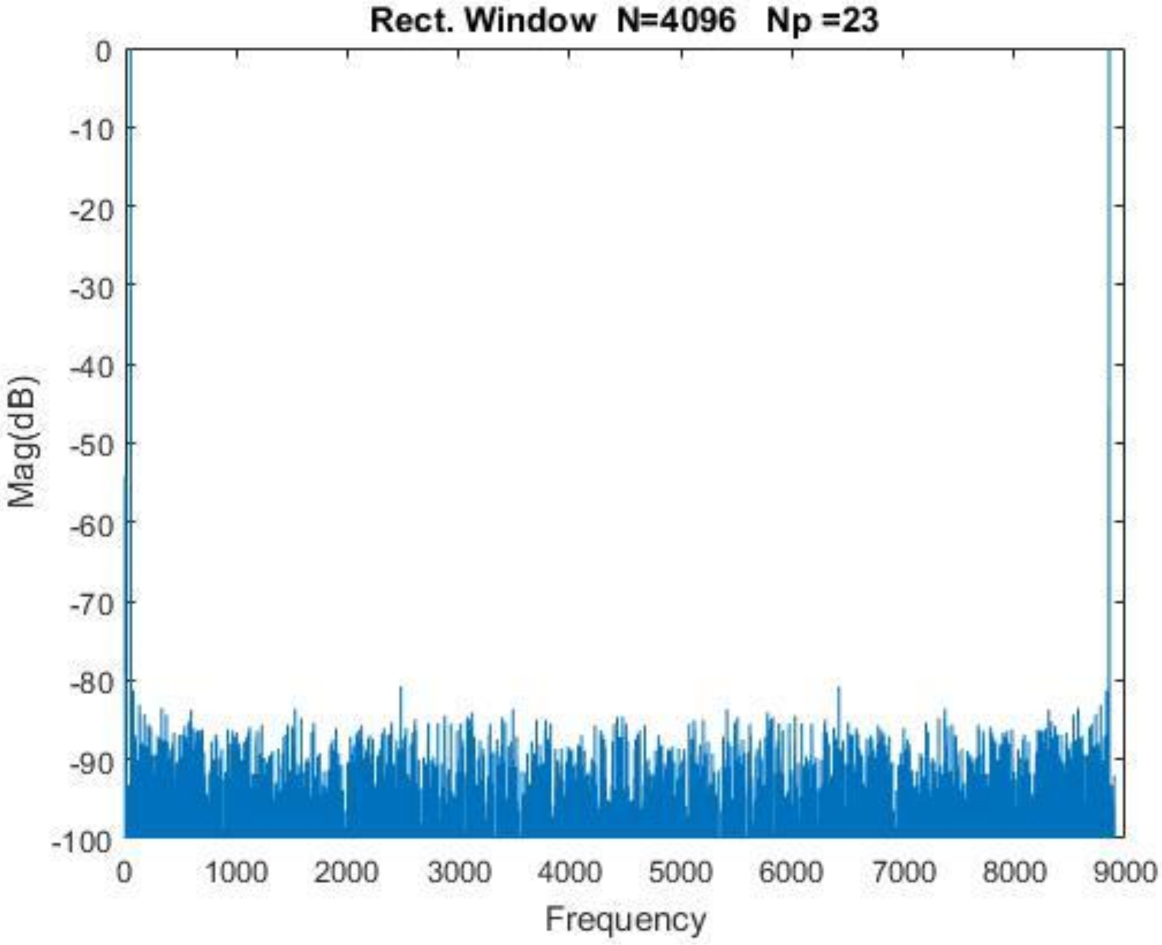
Res = 10 bits



Harmonic Components not Visible

Quantization Effects

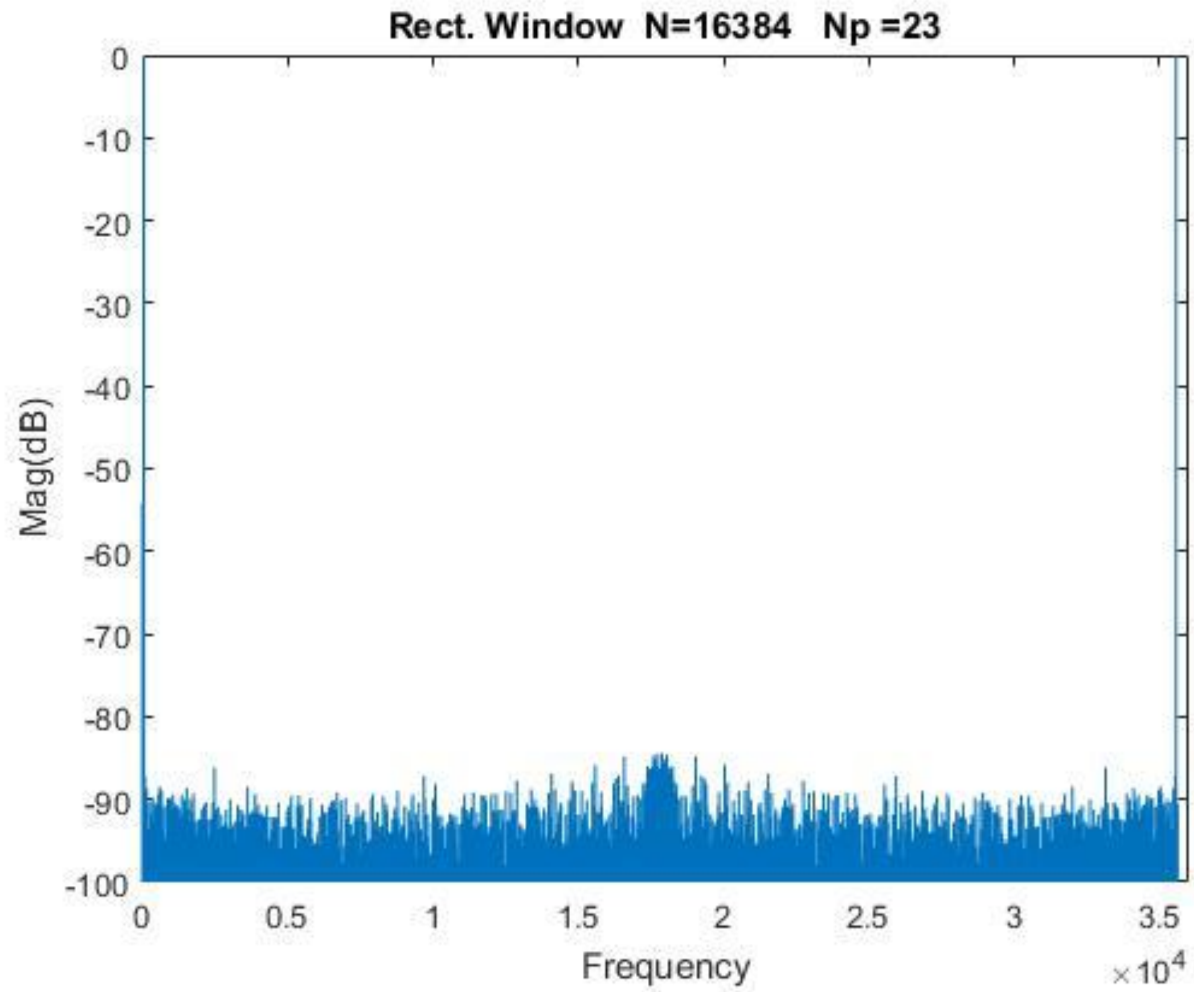
Res = 10 bits



Compared to the previous slide, it appears that the quantization noise has gone down – why does this occur?

Quantization Effects

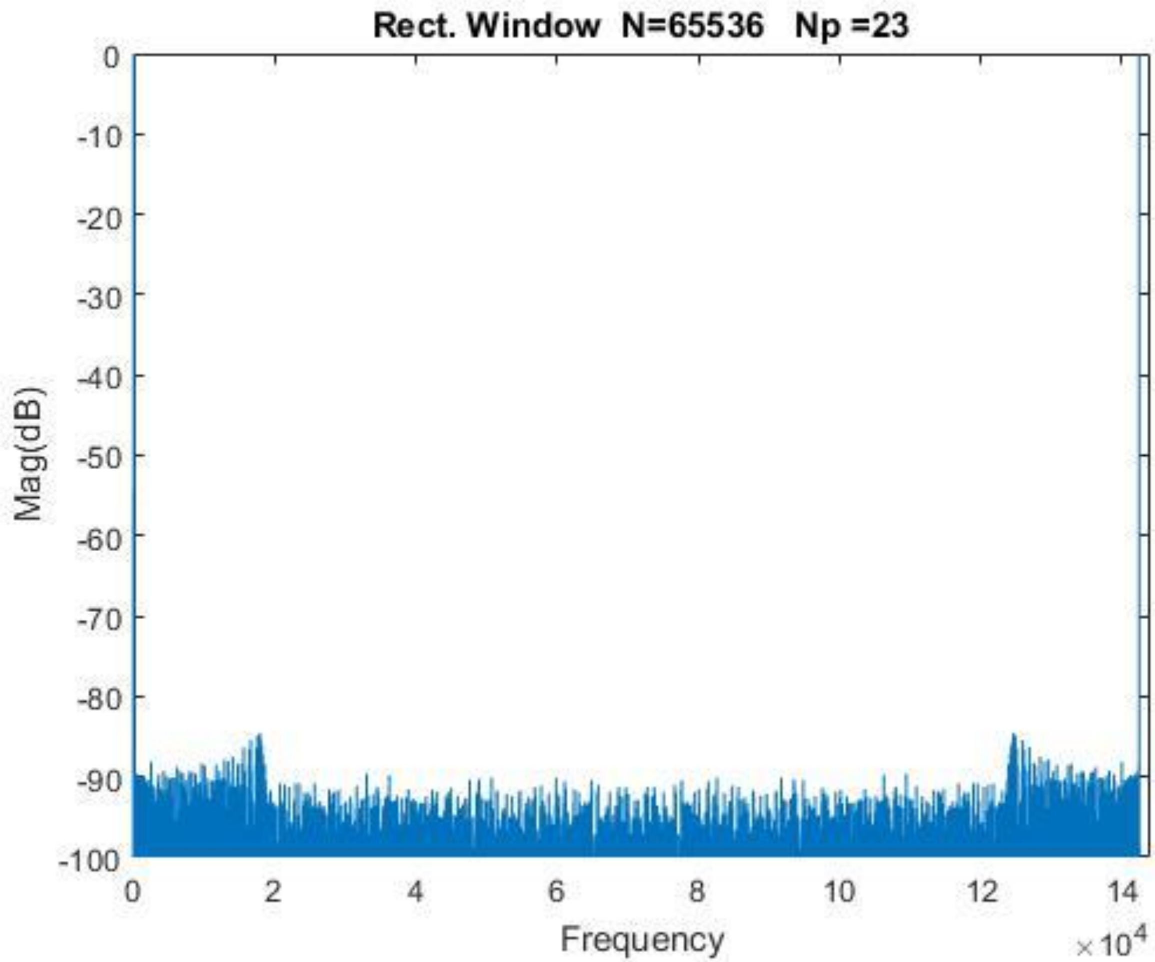
Res = 10 bits



Compared to the previous slide, it appears that the quantization noise has gone down even more – why does this occur?

Quantization Effects

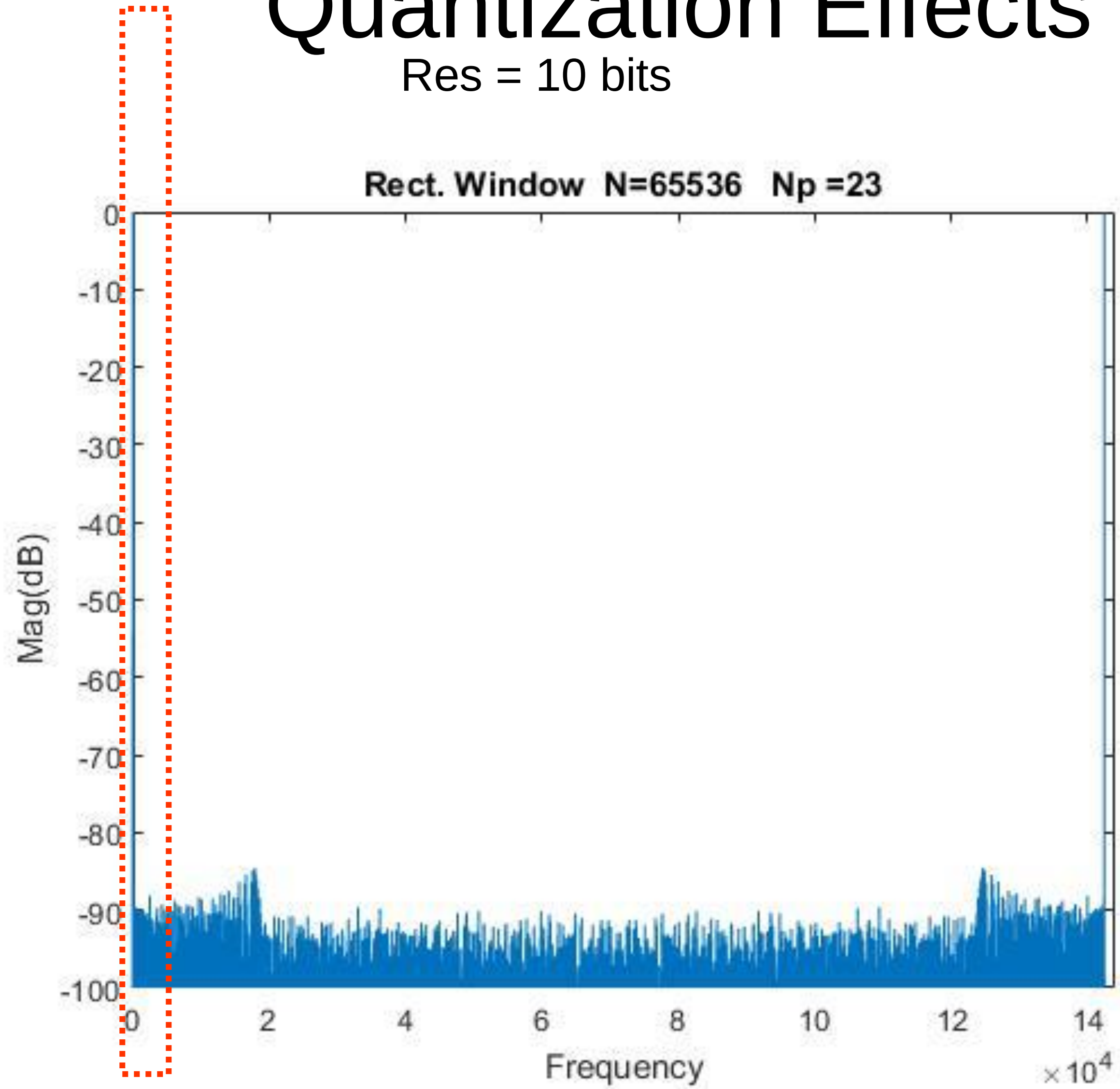
Res = 10 bits



Compared to the previous slides, it appears that the quantization noise has gone down even more – why does this occur?

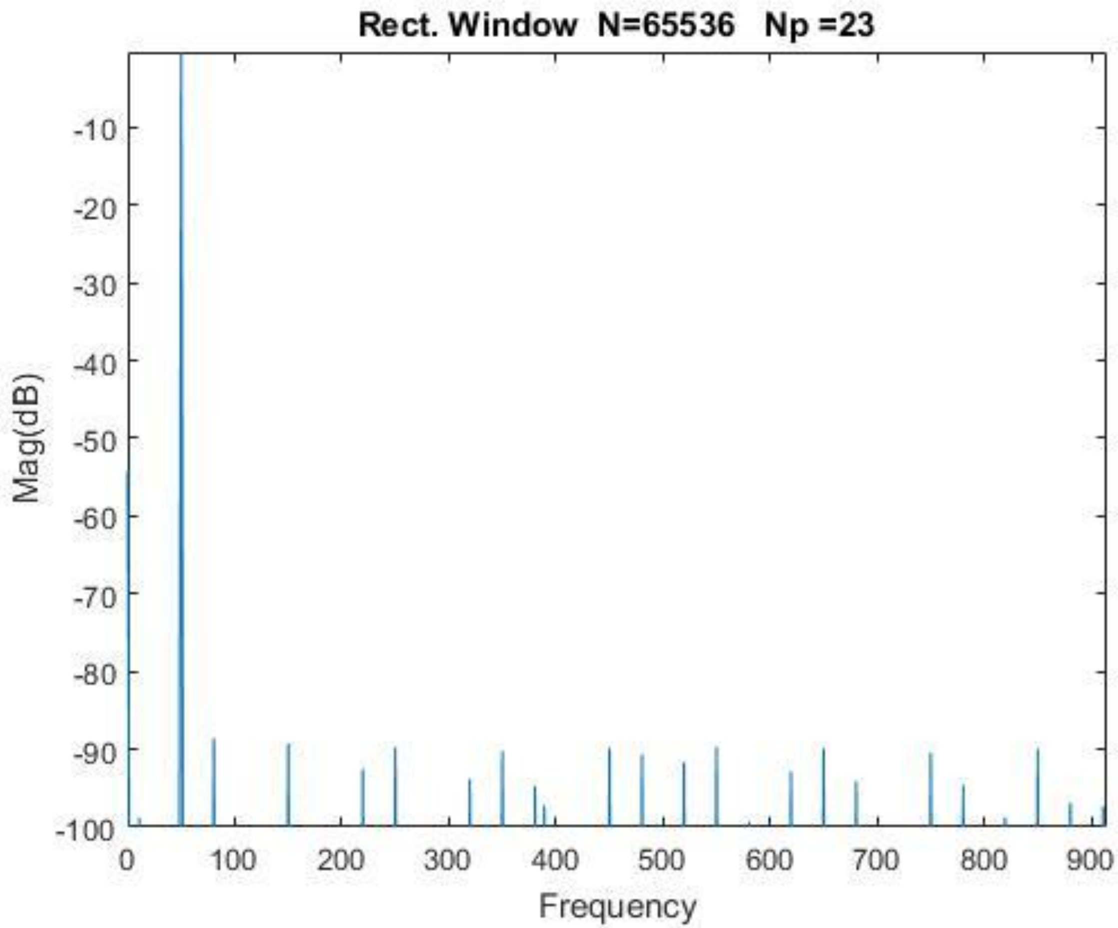
Quantization Effects

Res = 10 bits



Quantization Effects

Res = 10 bits



Very small third harmonic component but does not extend above other noise terms

Spectral Characterization

– Amplitude Quantization

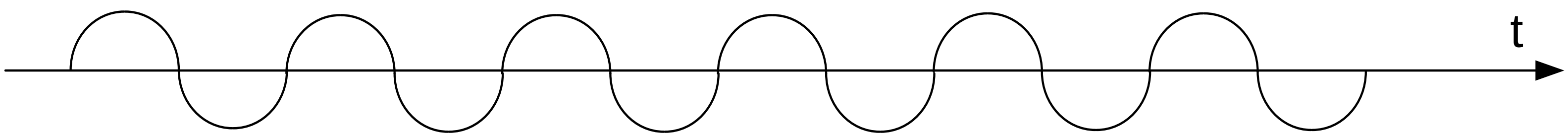
- Does not introduce substantive spectral components for n large
- Nearly uniformly distributed
- Decreases with increasing N

Spectral Characterization of Data Converters

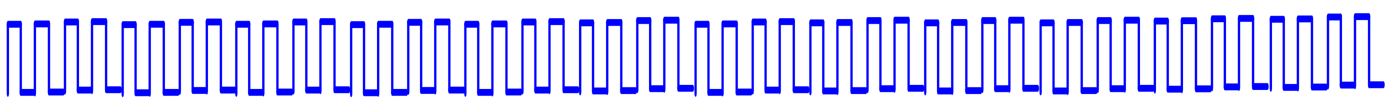
- Distortion Analysis
- Time Quantization Effects
  - of DACs
  - of ADCs
- Amplitude Quantization Effects
  - of DACs
  - of ADCs
- Clock Jitter

Spectral Characteristics of DACs and ADCs

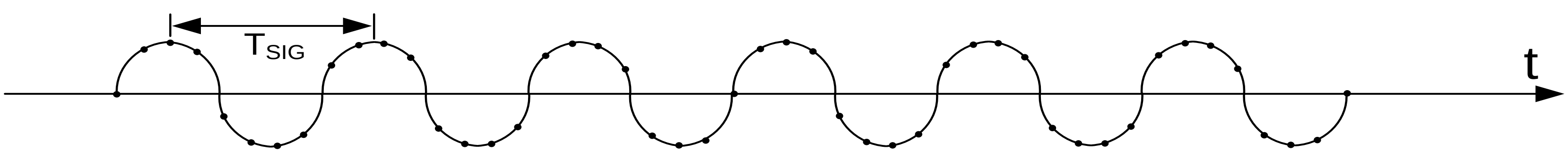
Spectral Characteristics of DAC



Periodic Input Signal

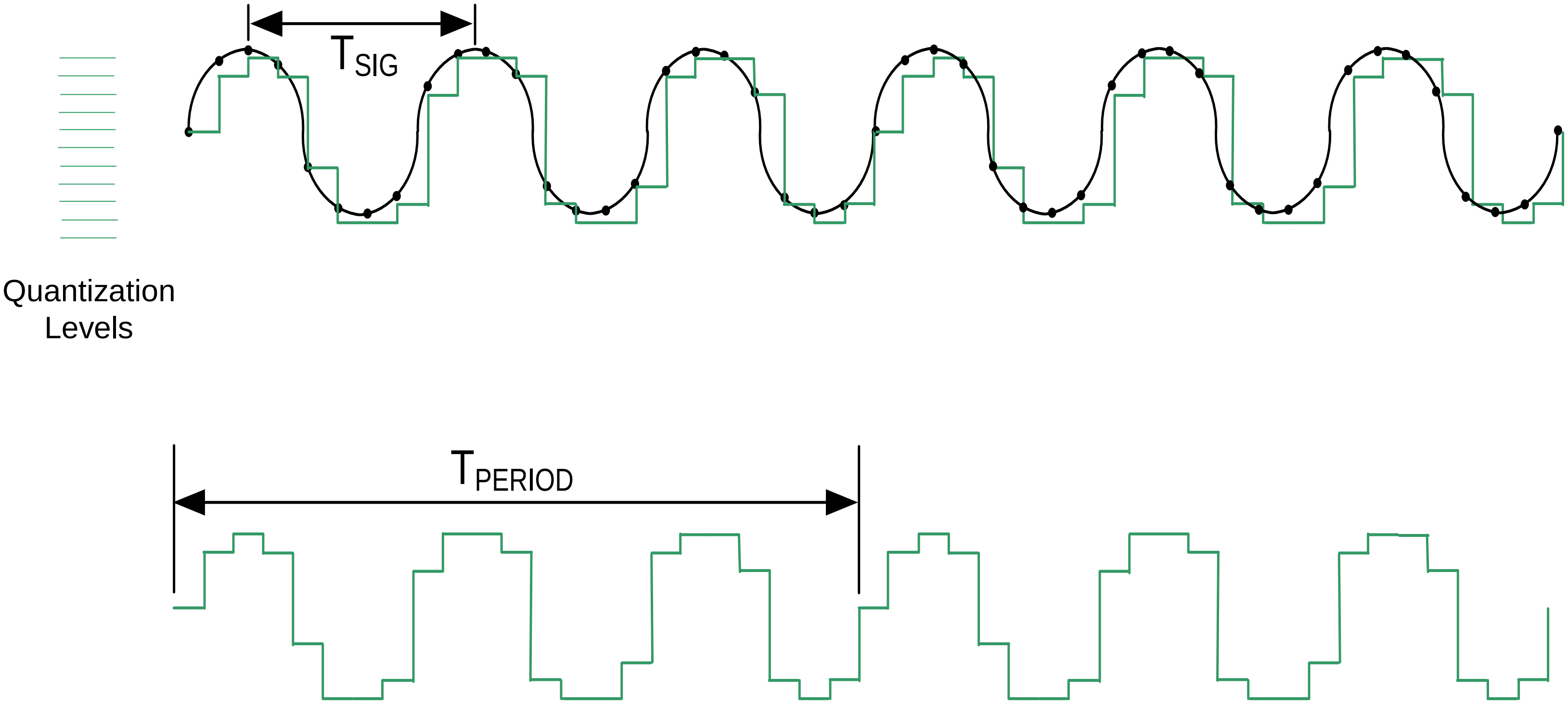


Sampling Clock



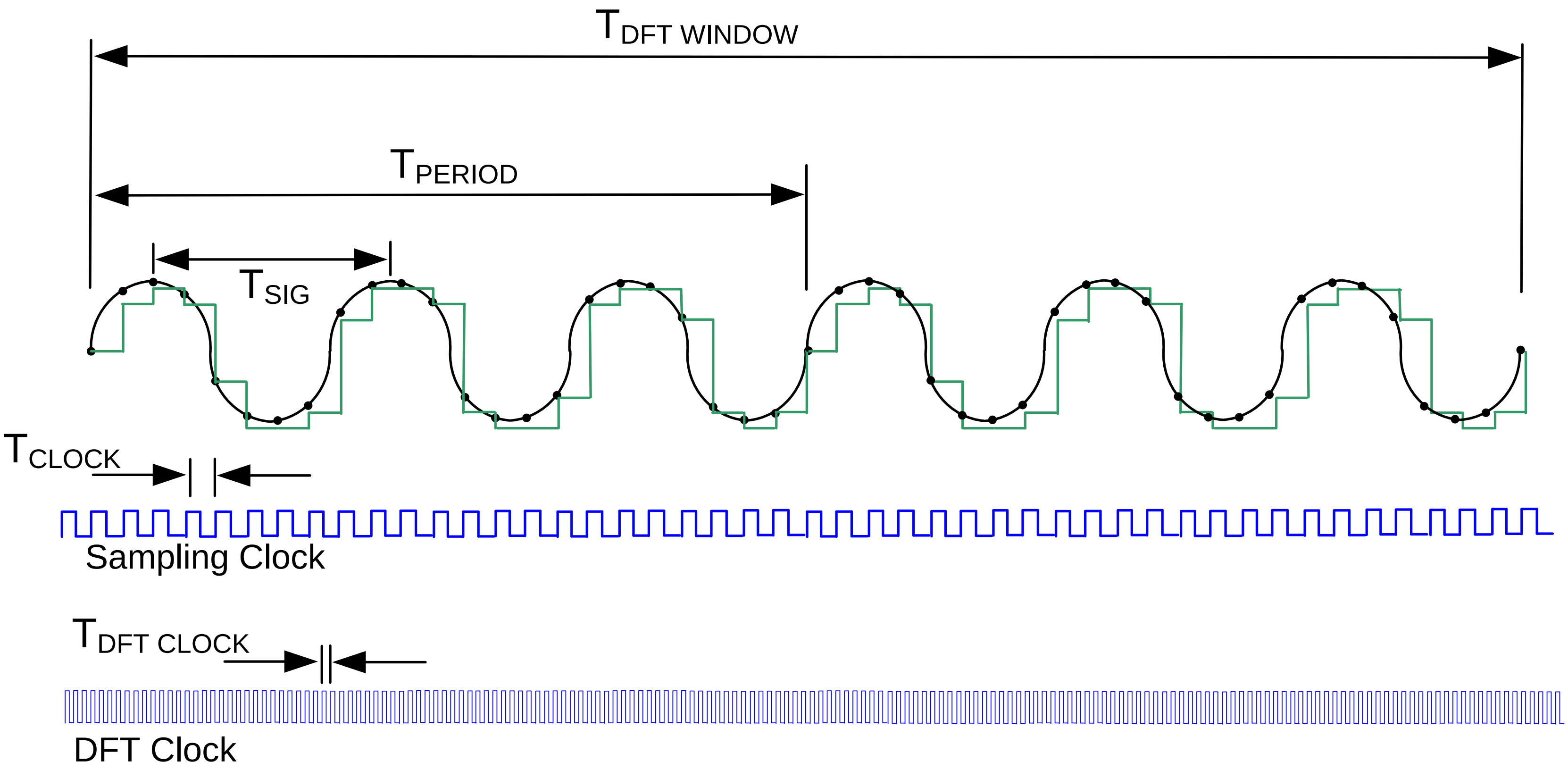
Sampled Input Signal (showing time points where samples taken)

Spectral Characteristics of DAC

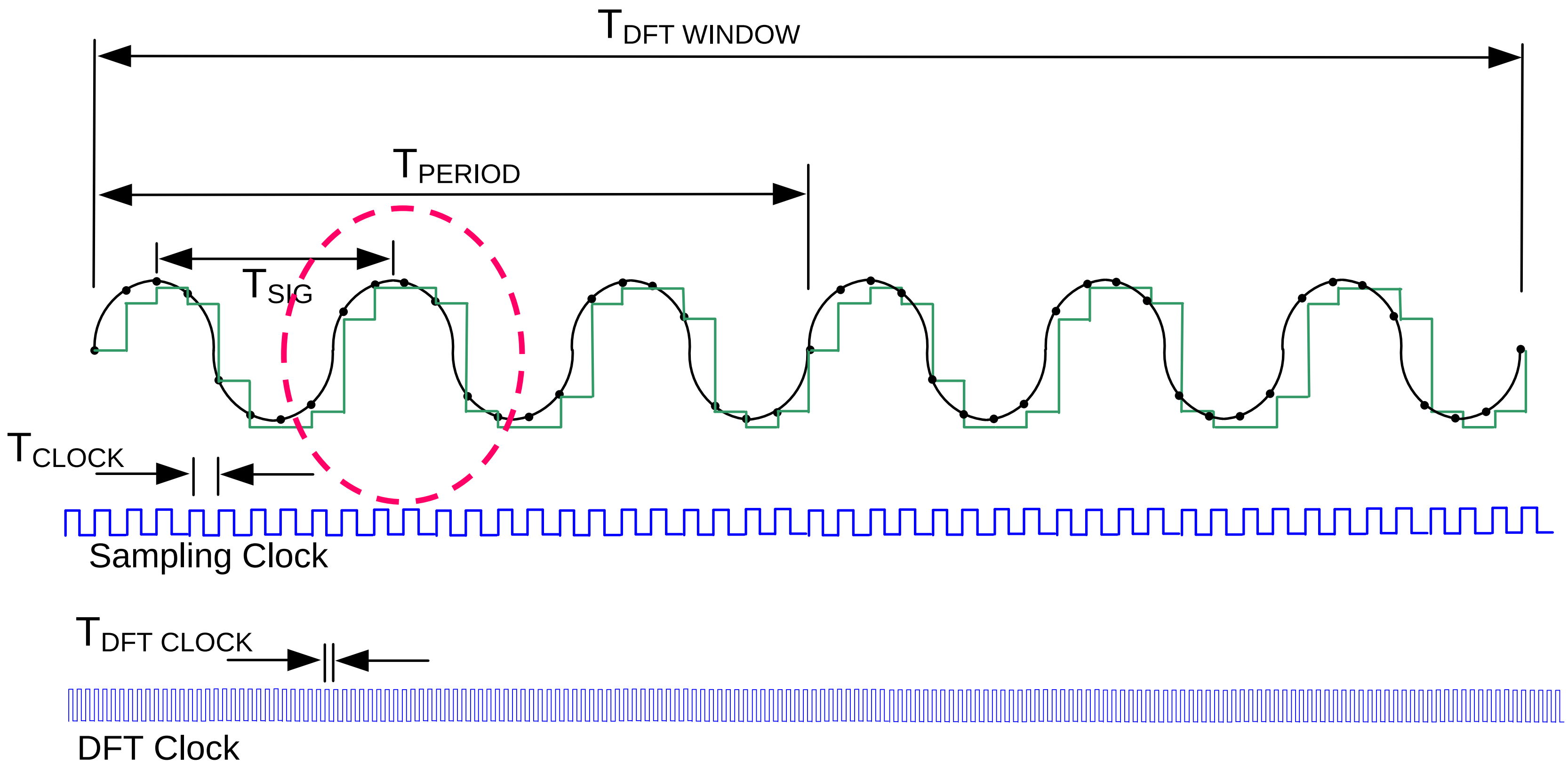


Quantized Sampled Input Signal (with zero-order sample and hold)

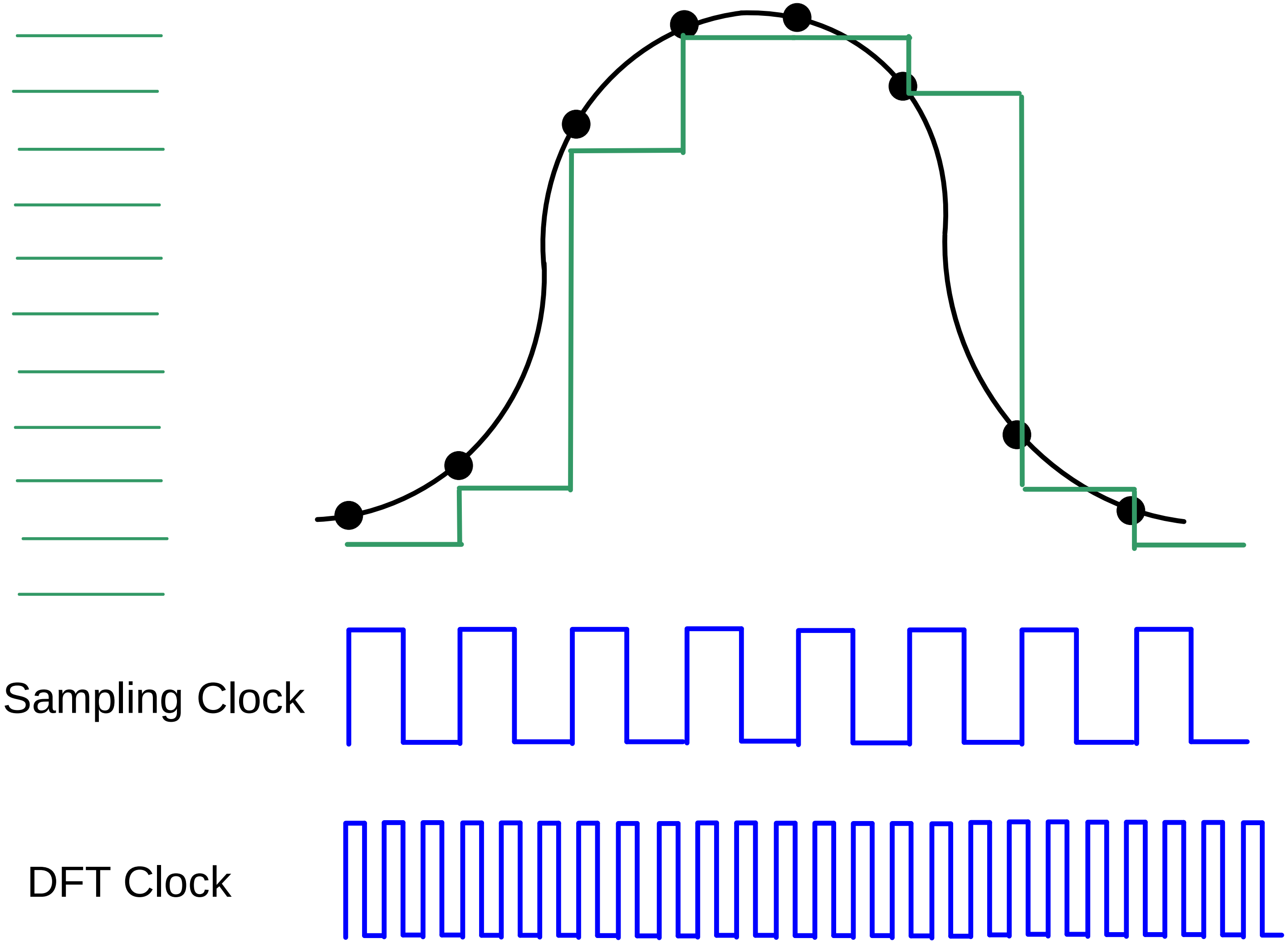
Spectral Characteristics of DAC



Spectral Characteristics of DAC

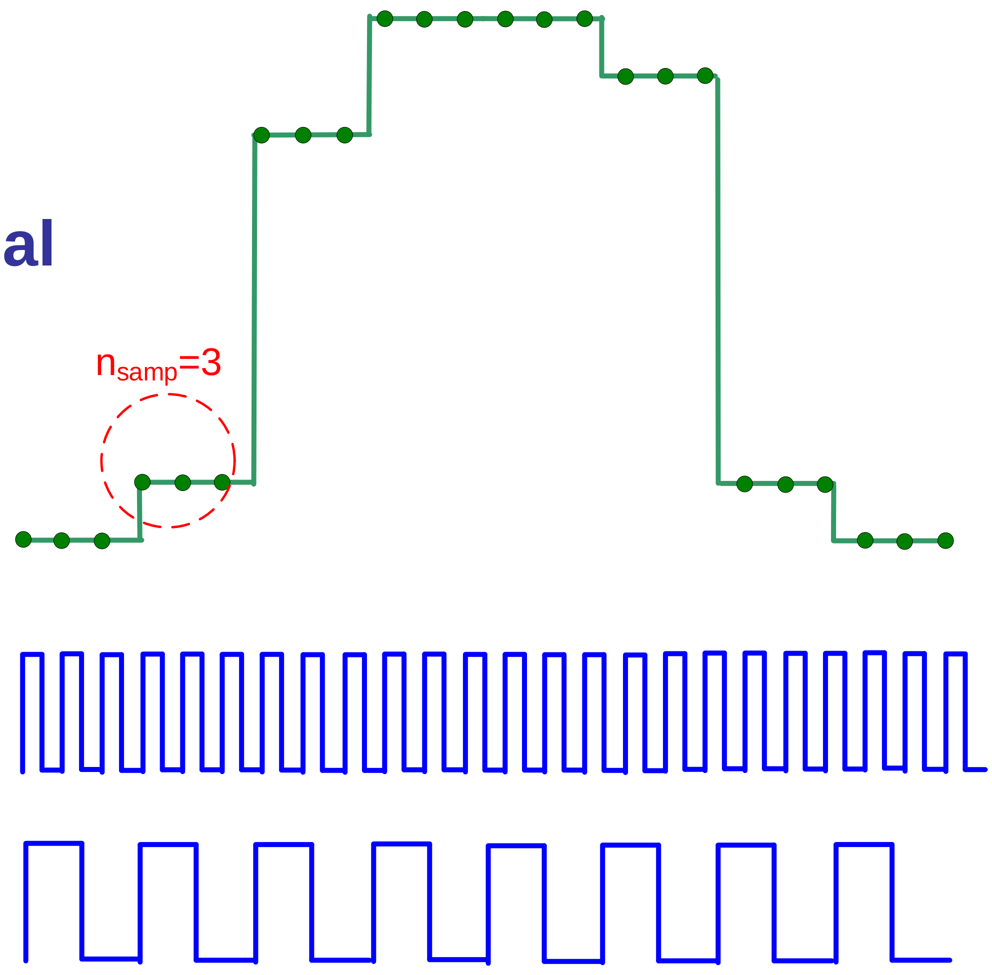


Spectral Characteristics of DAC



Spectral Characteristics of DAC

Sampled Quantized Signal (zoomed)



DFT Clock

Sampling Clock

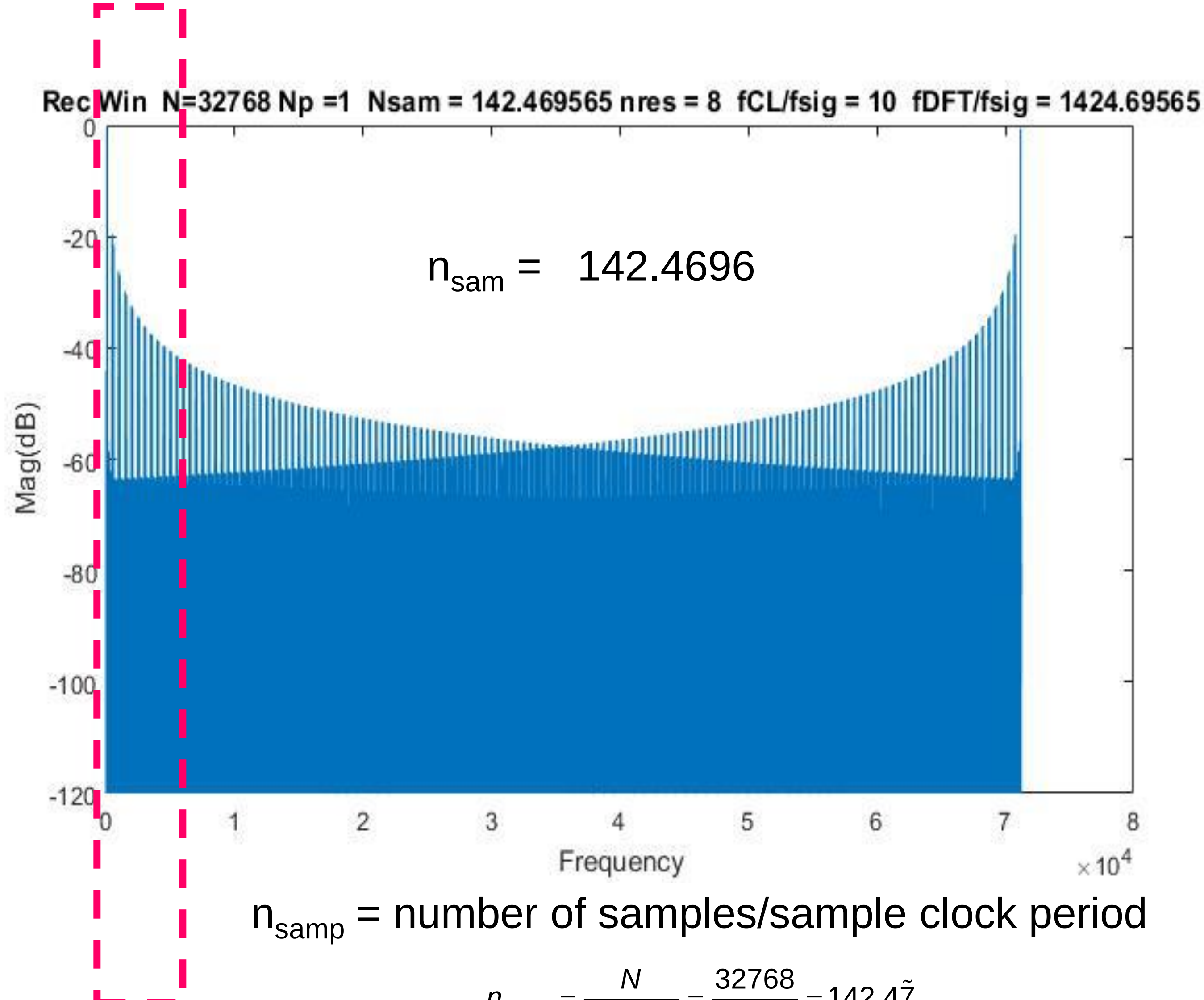
Spectral Characteristics of DAC

Consider the following example

- f_{SIG}=50 Hz
- f_{CL}=500 Hz (DAC clock)
- f_{DFTCL}=71.24K Hz (coherent sampling)
- n_{DFT}=15 $N = 2^{15} = 32,768$
- N_{P1}=23 (number of signal periods in DFT window)
- N_P=1
- n_{res}=8 bits
- $Xin(t) = .95\sin(2\pi f_{SIG}t)$ (-.4455dB)

Matlab File: afft_Quantization_DAC_Jan2017.m

DFT Simulation from Matlab

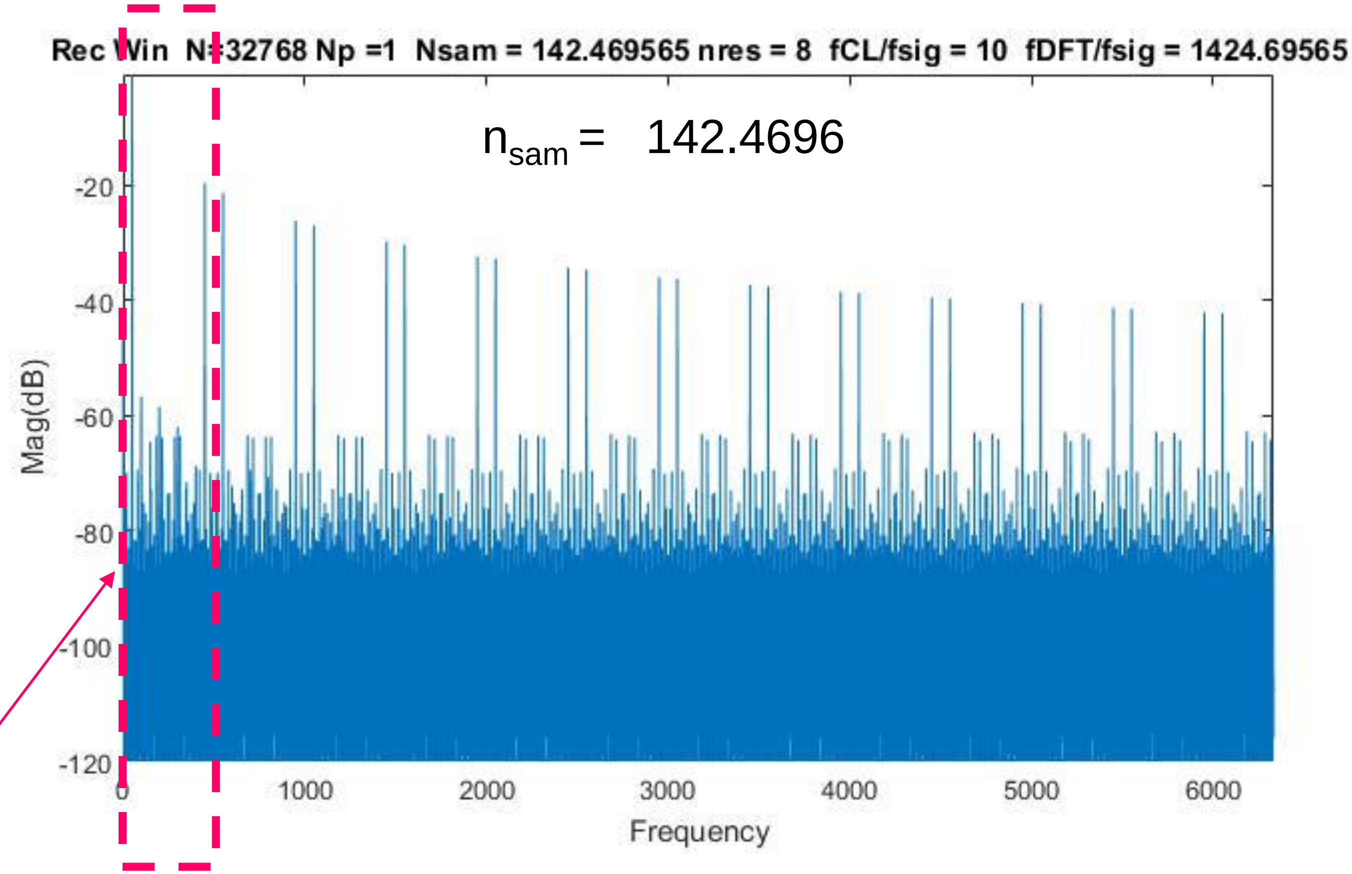


n_{samp} = number of samples/sample clock period

$$n_{SAMP} = \frac{N}{N_{P1}\frac{f_{CLK}}{f_{SIG}}} = \frac{32768}{23 \bullet 10} = 142.4\tilde{7}$$

DFT Simulation from Matlab

Expanded View

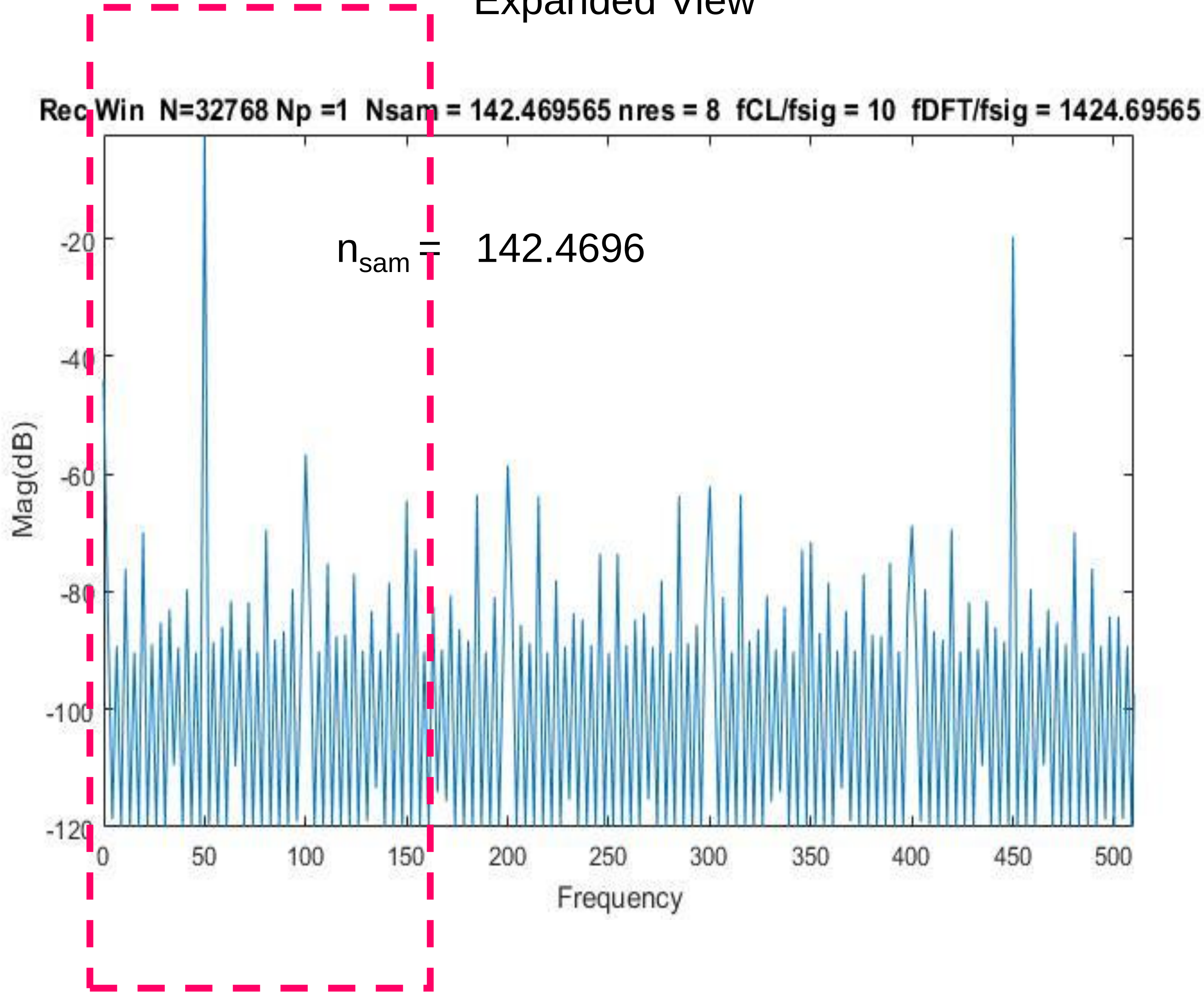


Width of this region is f_{CL}

Analogous to the overall DFT window when directly sampled but modestly asymmetric

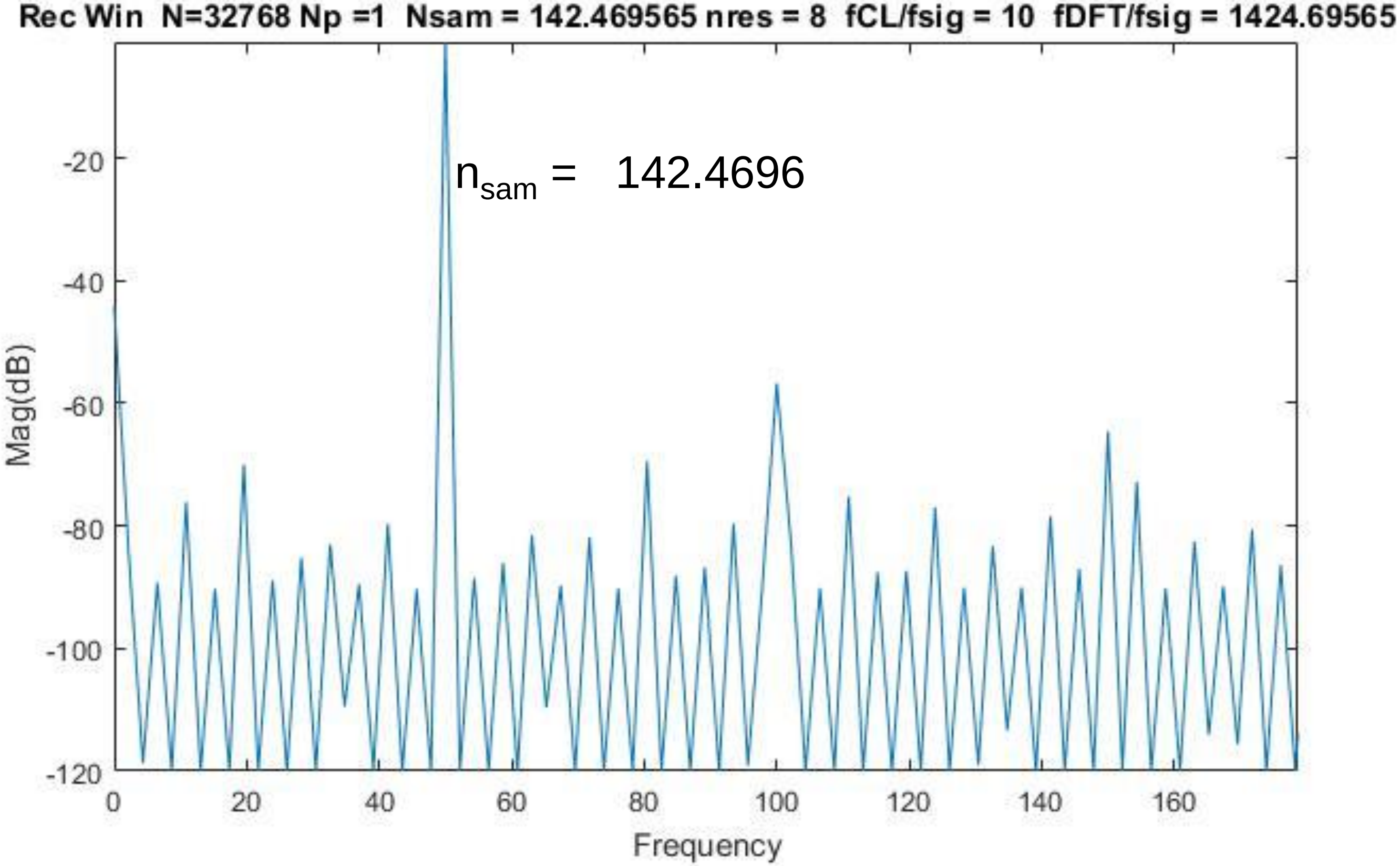
DFT Simulation from Matlab

Expanded View



DFT Simulation from Matlab

Expanded View



(2^{15}=32768)

DAC Comparisons with Quantization

Fundamental, second harmonic, and third harmonic

```
  Columns 1 through 12

  -44.0825  -84.2069 -118.6751  -89.2265 -120.0000  -76.0893 -120.0000  -90.3321 -120.0000  -69.9163 -120.0000  -88.9097

  Columns 13 through 24

 -120.0000  -85.1896 -120.0000  -83.0183 -109.4722  -89.4980 -120.0000  -79.6110 -120.0000  -90.2992 -120.0000   -0.5960

  Columns 25 through 36

 -120.0000  -88.5446 -120.0000  -86.0169 -120.0000  -81.5409 -109.6386  -89.7275 -120.0000  -81.8340 -120.0000  -90.2331

  Columns 37 through 48

 -120.0000  -69.4356 -120.0000  -88.1400 -120.0000  -86.7214 -120.0000  -79.6273 -119.1428  -89.9175  -56.7024  -83.0511

  Columns 49 through 60

 -120.0000  -90.1331 -120.0000  -75.1821 -120.0000  -87.5706 -120.0000  -87.3205 -120.0000  -76.9769 -120.0000  -90.0703

  Columns 61 through 72

 -119.0588  -83.2950 -113.3964  -89.9982 -120.0000  -78.4288 -120.0000  -87.0328 -120.0000  -64.5409 -120.0000  -72.8111

  Columns 73 through 84

 -120.0000  -90.1876 -120.0000  -82.5616 -114.0867  -89.8269 -115.6476  -80.6553 -120.0000  -86.3818 -120.0000  -88.3454

  Columns 85 through 96

 -120.0000  -63.5207 -120.0000  -90.2704 -120.0000  -80.8524 -120.0000  -89.6174  -58.5435  -82.3253 -120.0000  -85.6188
```

N	θ	Nsam	n	A_1	A_2	A_3
32K	1	142.5	8	-.596	-56.7	-64.5
128K	1	569.9	8	-.596	-56.7	-64.45

Spectral Characteristics of DAC
(amplitude and time quantization)

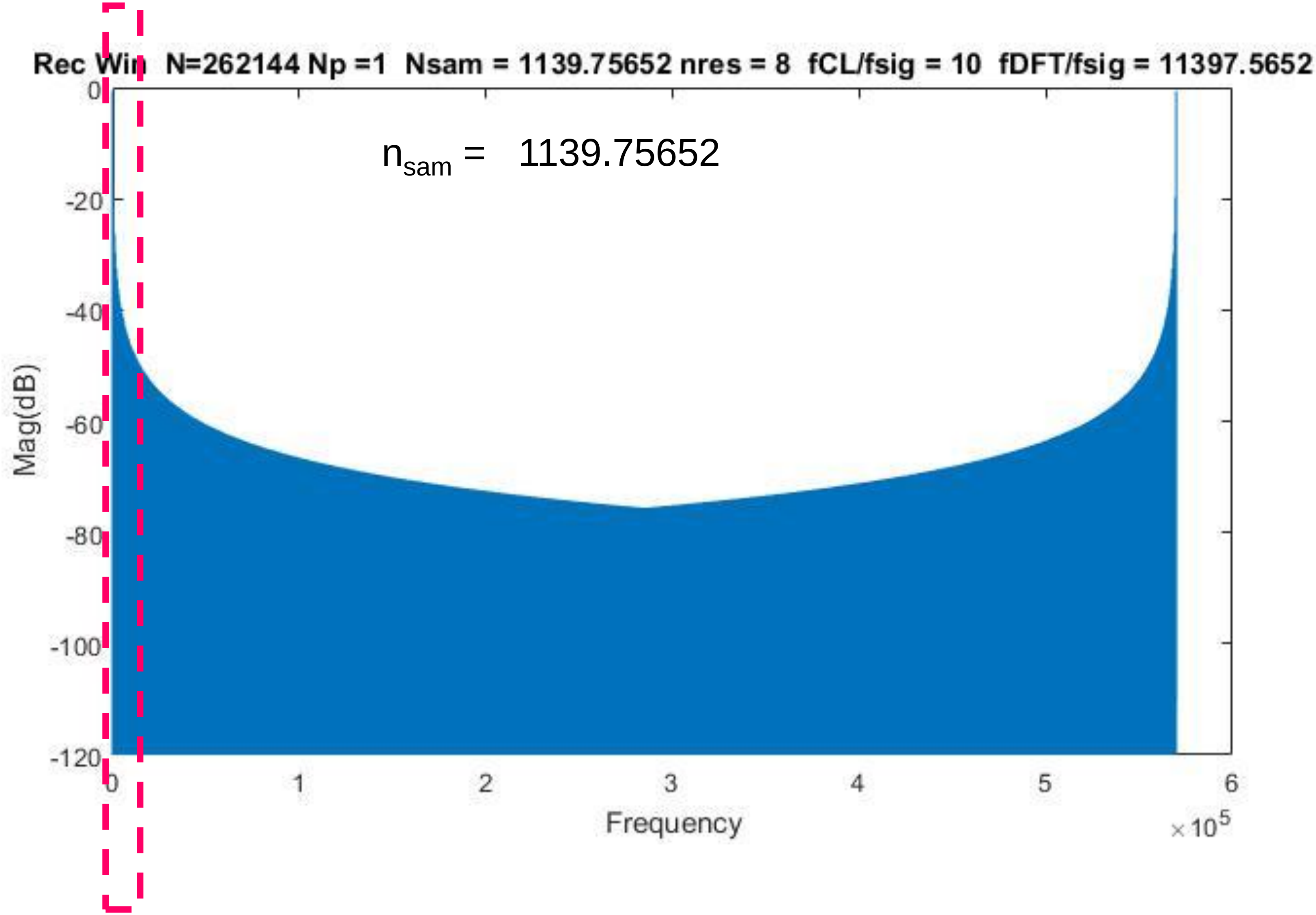
Consider the following example

- f_{SIG}=50 Hz
- f_{CL}=500 Hz (DAC clock)
- f_{DFTCL}=71.24K Hz (coherent sampling)
- n_{DFT}=18 $N = 2^{18} = 262{,}144$
- N_{P1}=23 (number of signal periods in DFT window)
- N_P=1
- n_{res}=8 bits
- Xin(t) =.95sin($2\pi f_{SIG}$t) (-.4455dB)

increased →

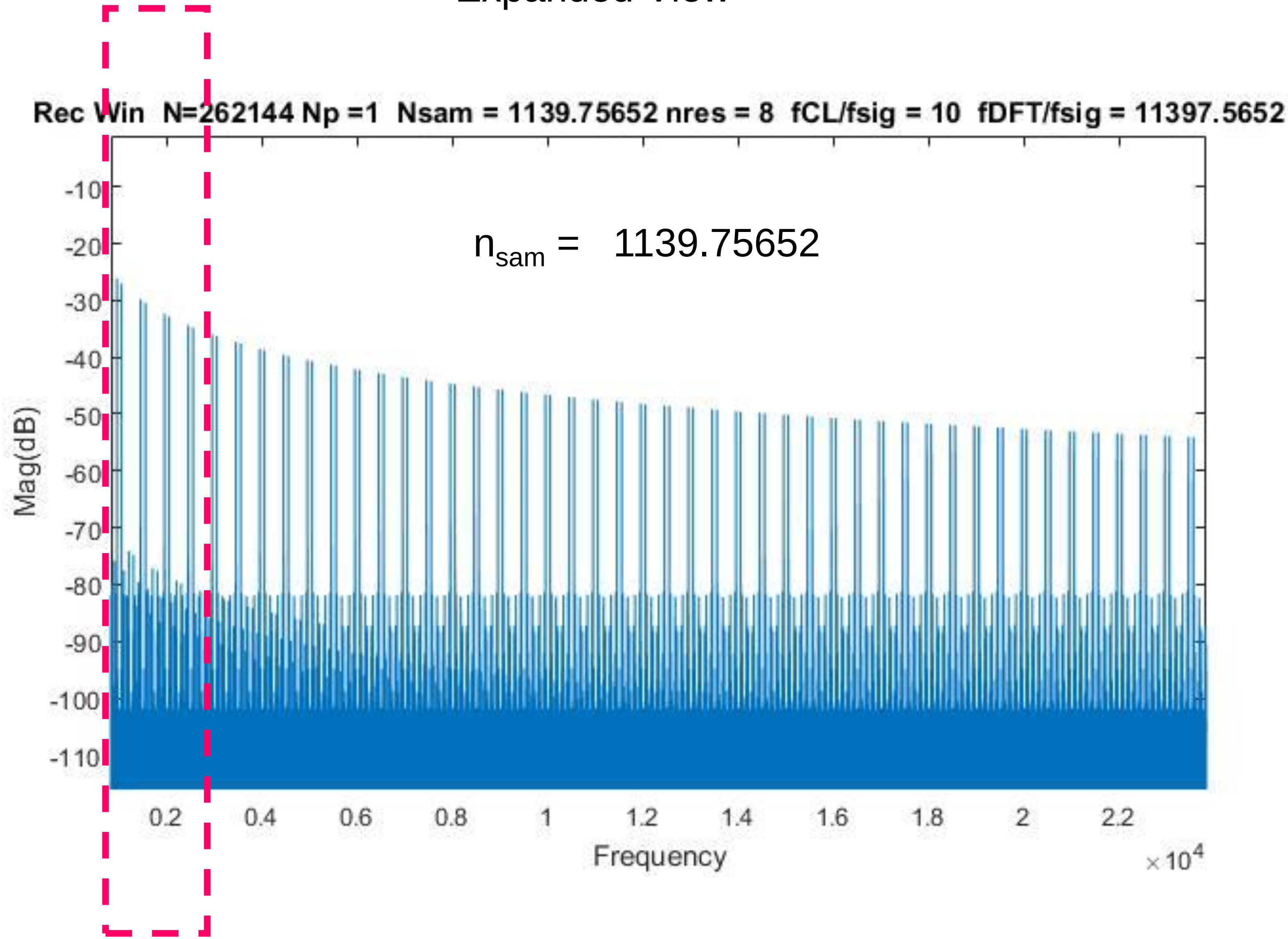
Matlab File: afft_Quantization_DAC.m

DFT Simulation from Matlab



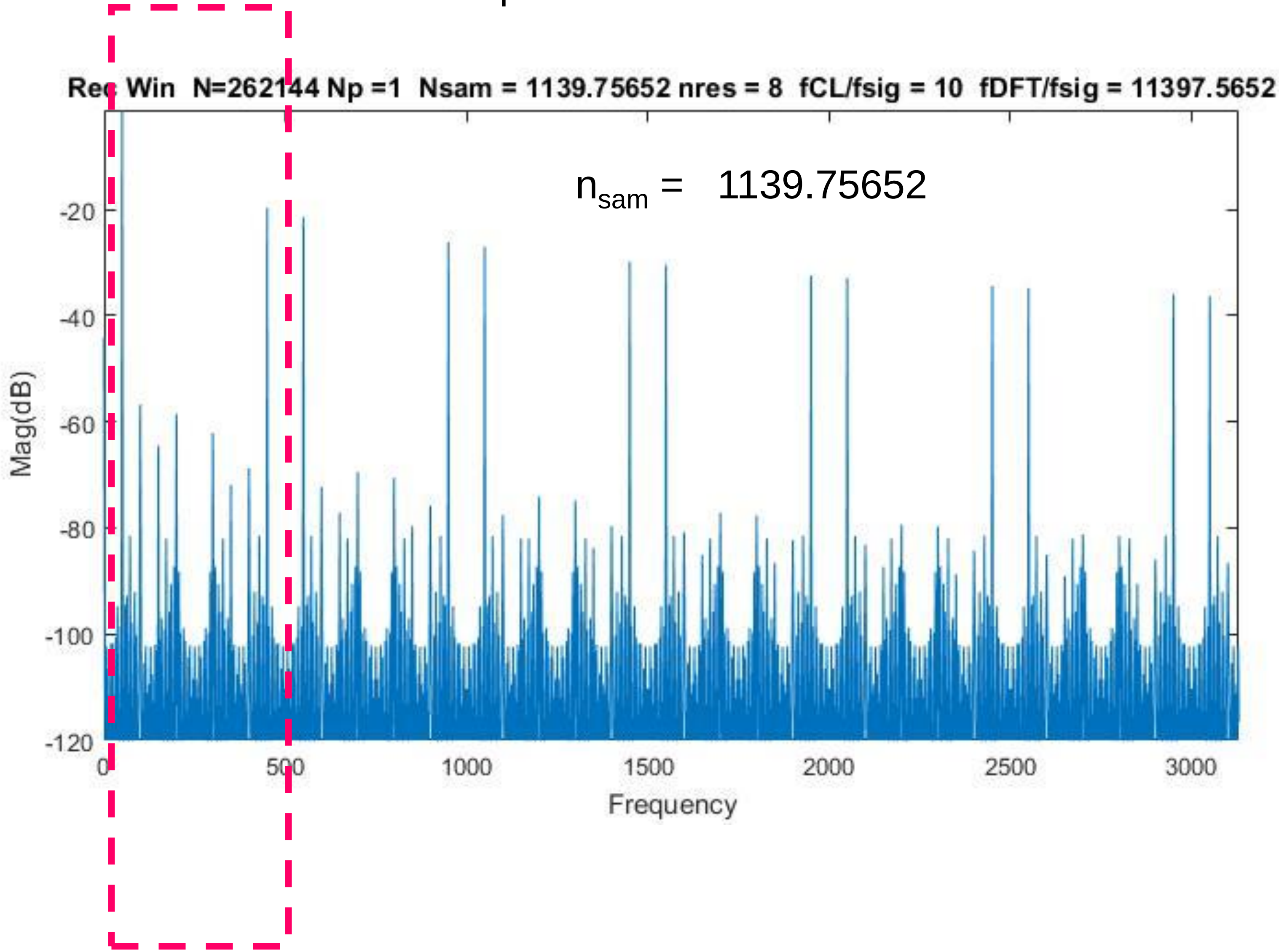
DFT Simulation from Matlab

Expanded View



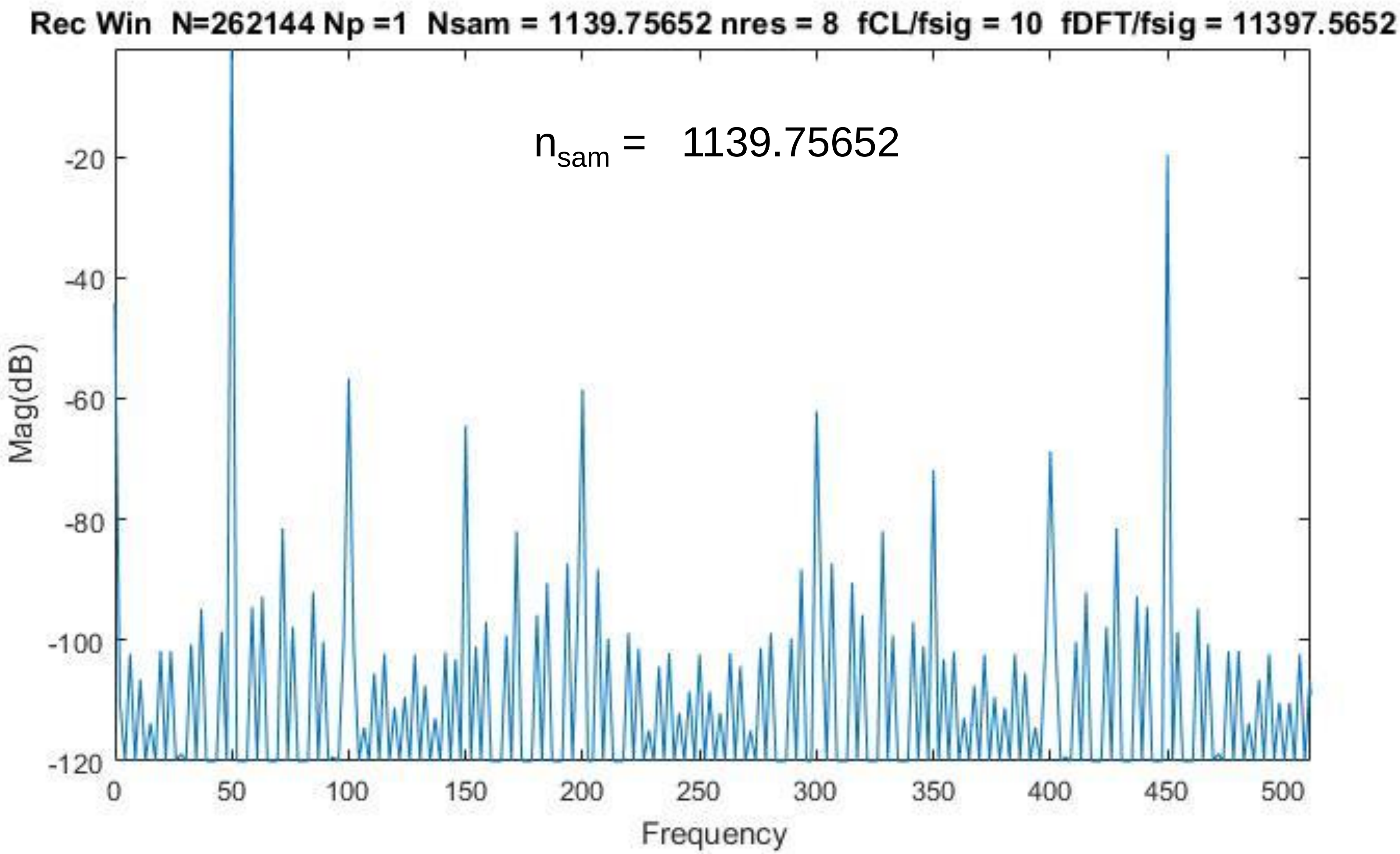
DFT Simulation from Matlab

Expanded View



DFT Simulation from Matlab

Expanded View

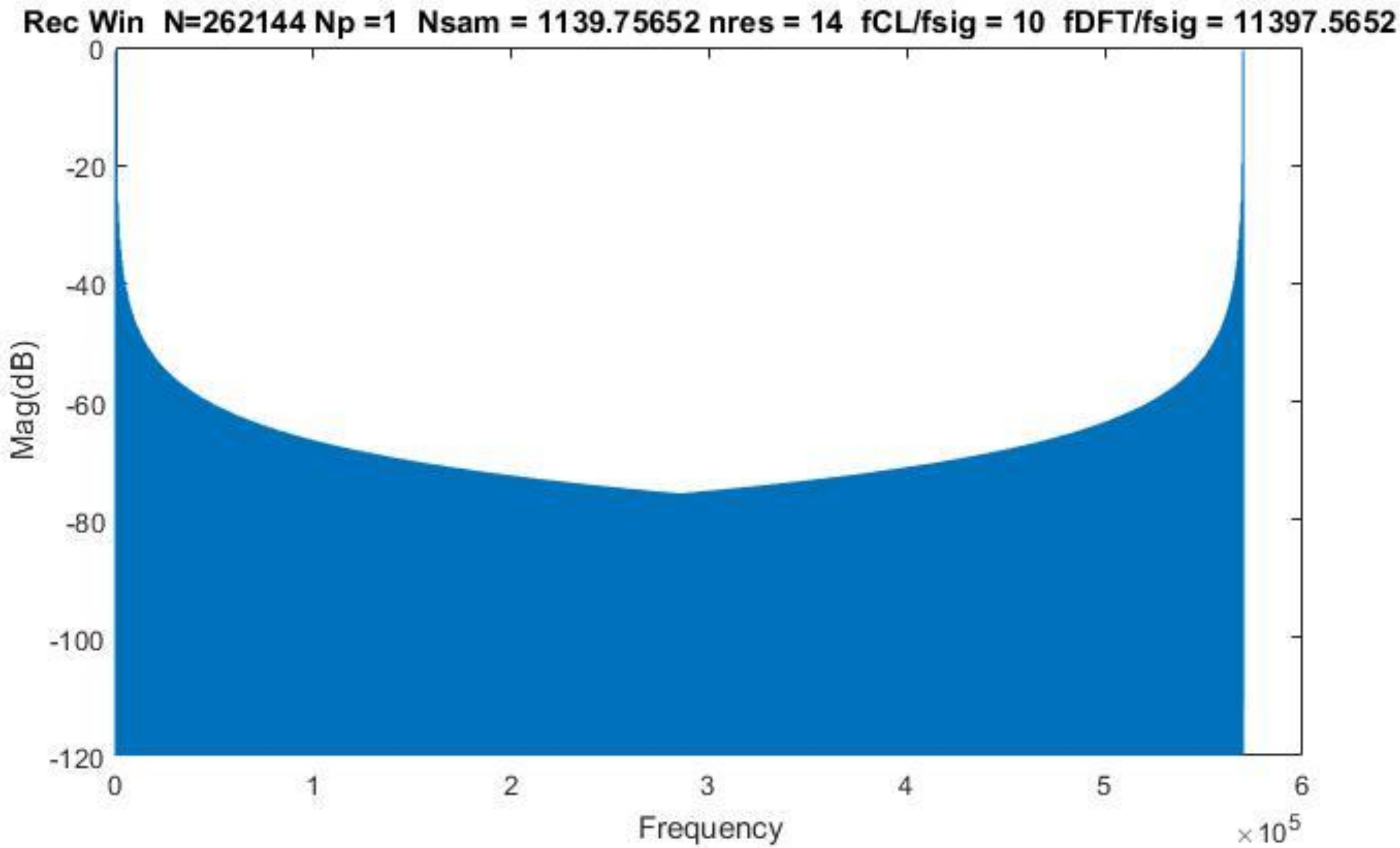


Spectral Characteristics of DAC

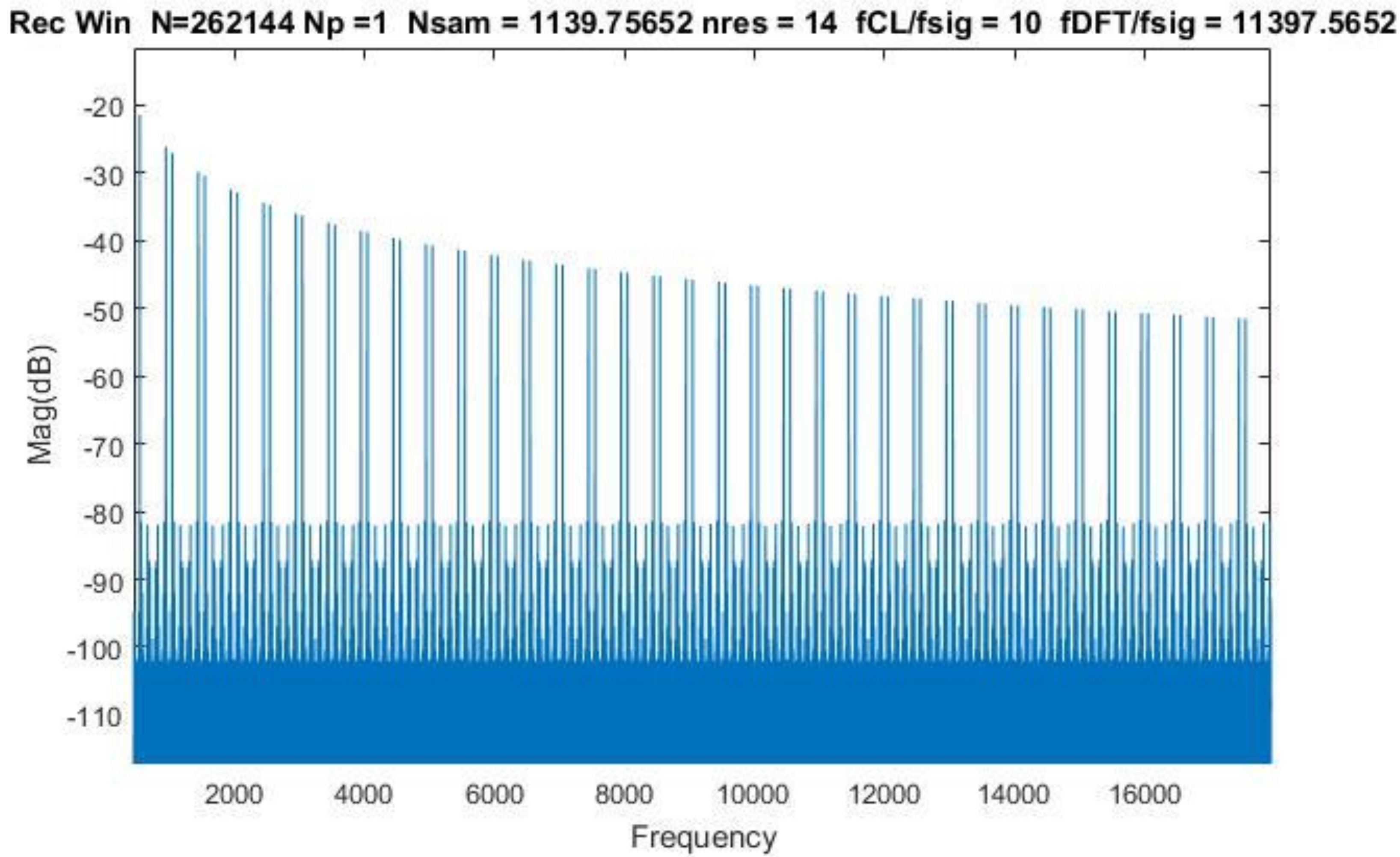
Consider the following example

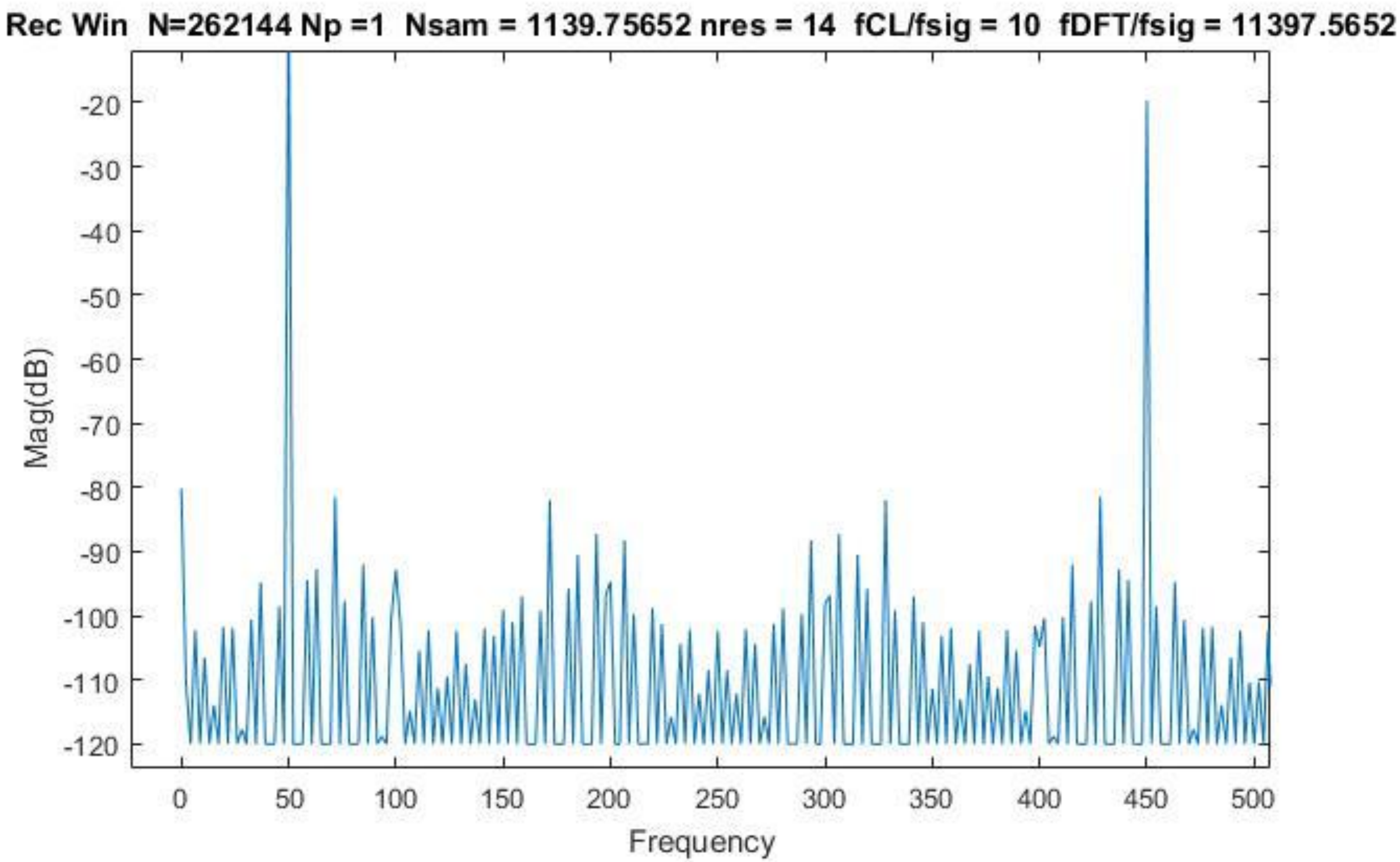
- f_{SIG}=50 Hz
- f_{CL}=500 Hz (DAC clock)
- f_{DFTCL}=71.24K Hz (coherent sampling)
- n_{DFT}=18
- N_{P1}=23 (number of signal periods in DFT window)
- N_P=1
- increased → n_{res}=14bits
- $Xin(t) = .95\sin(2\pi f_{SIG} t)$ (-.4455dB)

Matlab File: afft_Quantization_DAC.m



Rec Win N=262144 Np =1 Nsam = 1139.75652 nres = 14 fCL/fsig = 10 fDFT/fsig = 11397.5652
0
-20
-40
-60
-80
-100
-120
Mag(dB)
0
1
2
3
4
5
6
Frequency
×10^5


Rec Win N=262144 Np =1 Nsam = 1139.75652 nres = 14 fCL/fsig = 10 fDFT/fsig = 11397.5652
Mag(dB)
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
2000
4000
6000
8000
10000
12000
14000
16000
Frequency


Rec Win N=262144 Np =1 Nsam = 1139.75652 nres = 14 fCL/fsig = 10 fDFT/fsig = 11397.5652
Mag(dB)
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
0
50
100
150
200
250
300
350
400
450
500
Frequency

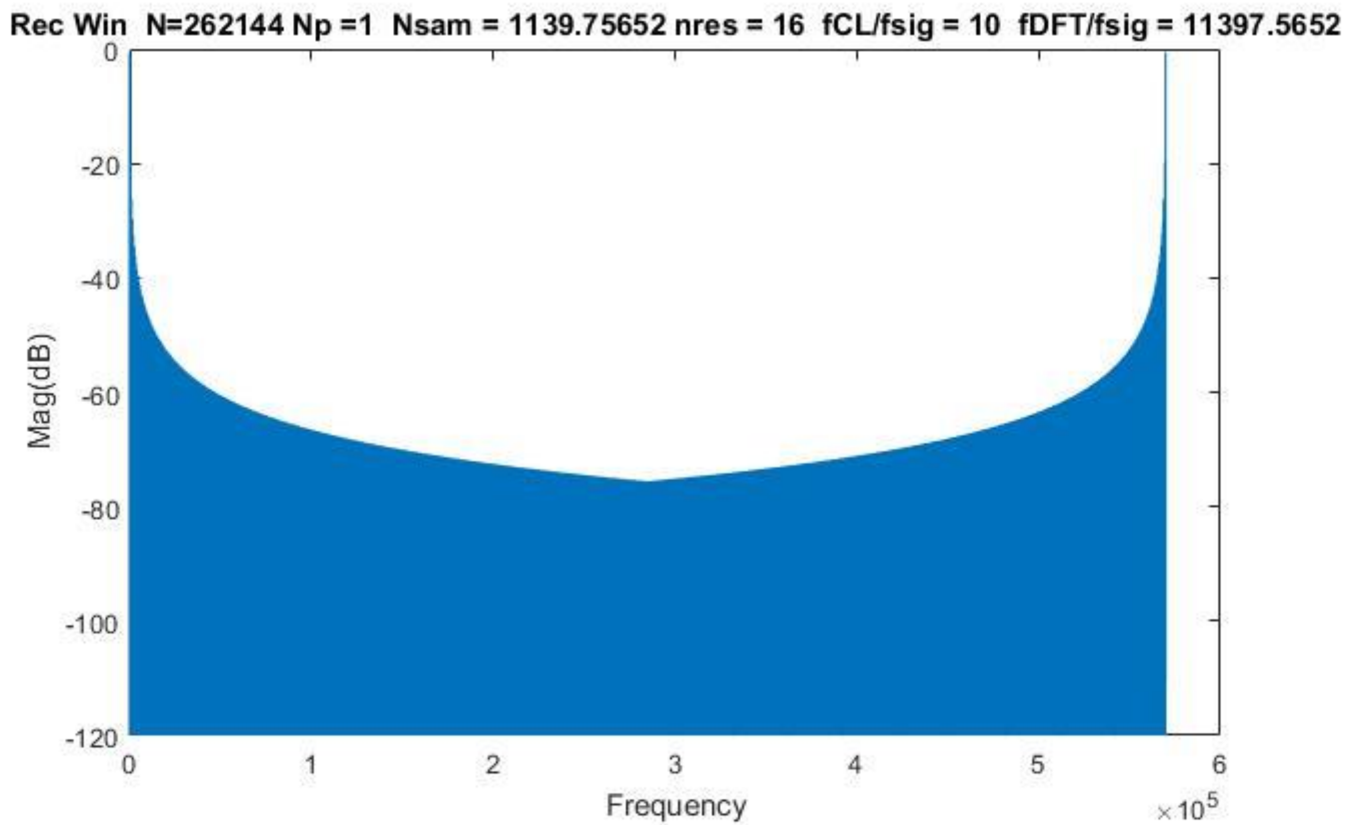
Spectral Characteristics of DAC

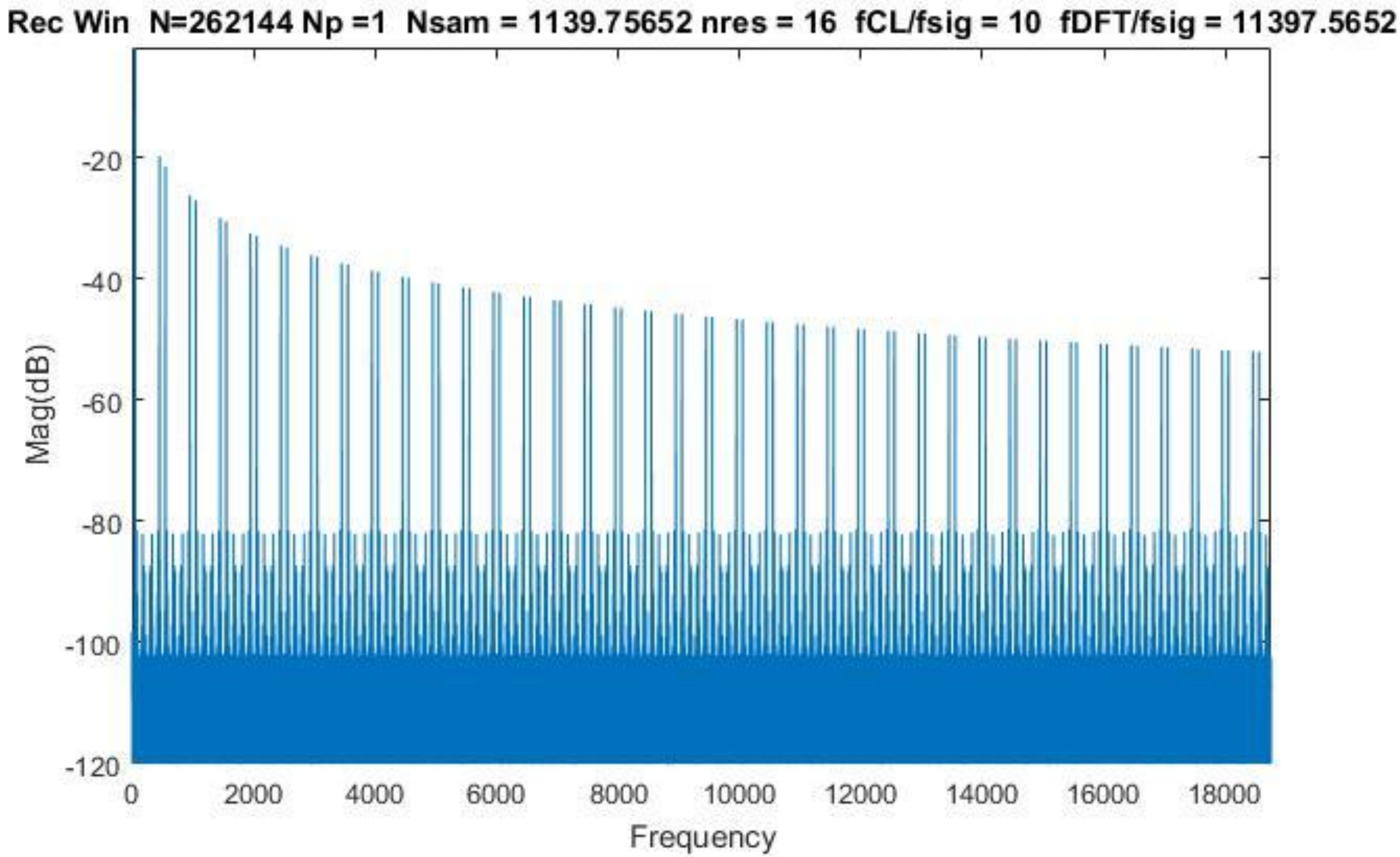
Consider the following example

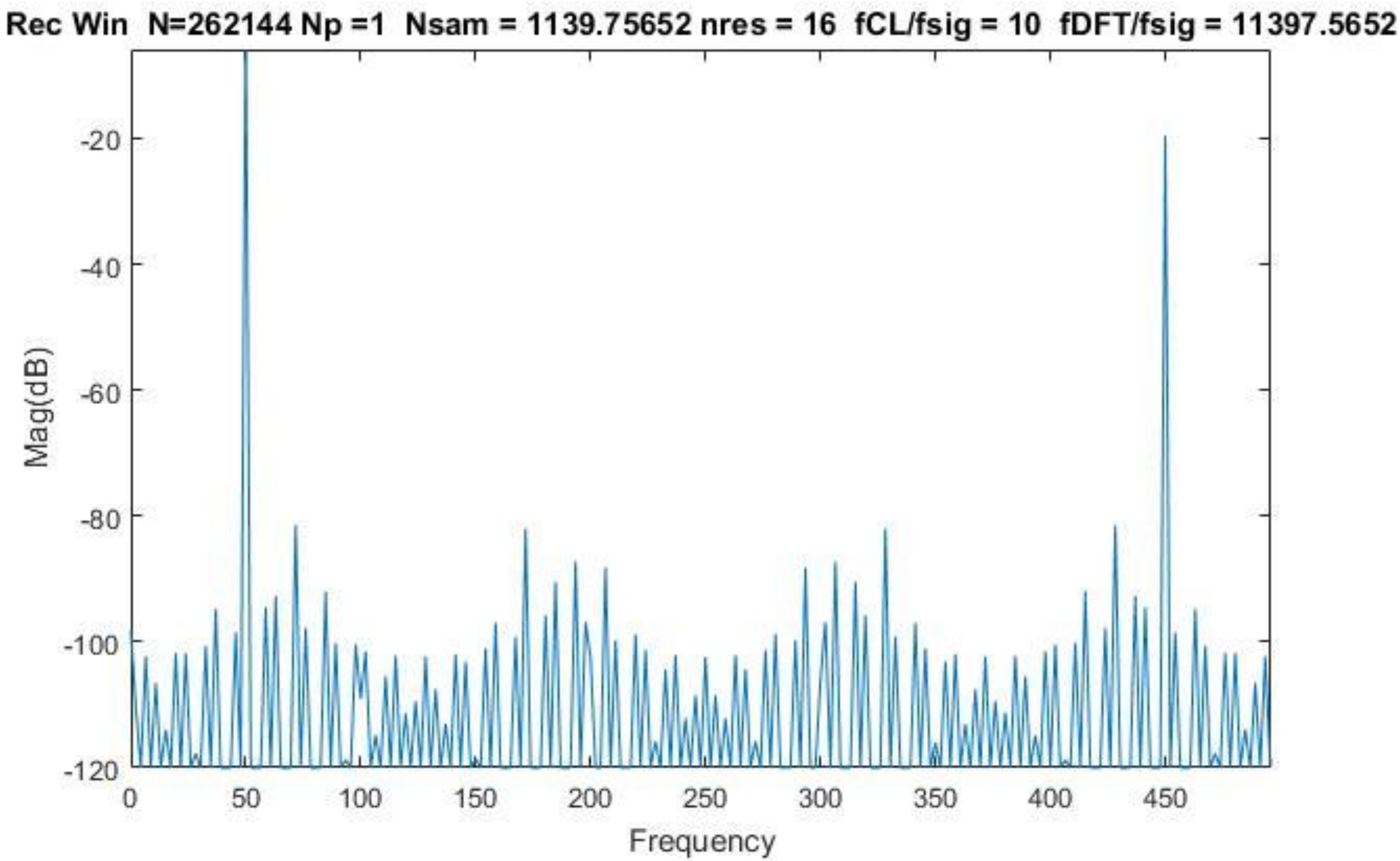
- f_{SIG}=50 Hz
- f_{CL}=500 Hz (DAC clock)
- f_{DFTCL}=71.24K Hz (coherent sampling)
- n_{DFT}=18
- N_{P1}=23 (number of signal periods in DFT window)
- N_P=1
- n_{res}=16bits
- Xin(t) =.95sin($2\pi f_{SIG}$t) (-.4455dB)

increased →

Matlab File: afft_Quantization_DAC.m


Rec Win N=262144 Np =1 Nsam = 1139.75652 nres = 16 fCL/fsig = 10 fDFT/fsig = 11397.5652
Mag(dB)
0
-20
-40
-60
-80
-100
-120
0
1
2
3
4
5
6
Frequency
×10^5


Rec Win N=262144 Np =1 Nsam = 1139.75652 nres = 16 fCL/fsig = 10 fDFT/fsig = 11397.5652
Mag(dB)
-20
-40
-60
-80
-100
-120
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
Frequency


Rec Win N=262144 Np =1 Nsam = 1139.75652 nres = 16 fCL/fsig = 10 fDFT/fsig = 11397.5652
Mag(dB)
-20
-40
-60
-80
-100
-120
0
50
100
150
200
250
300
350
400
450
Frequency

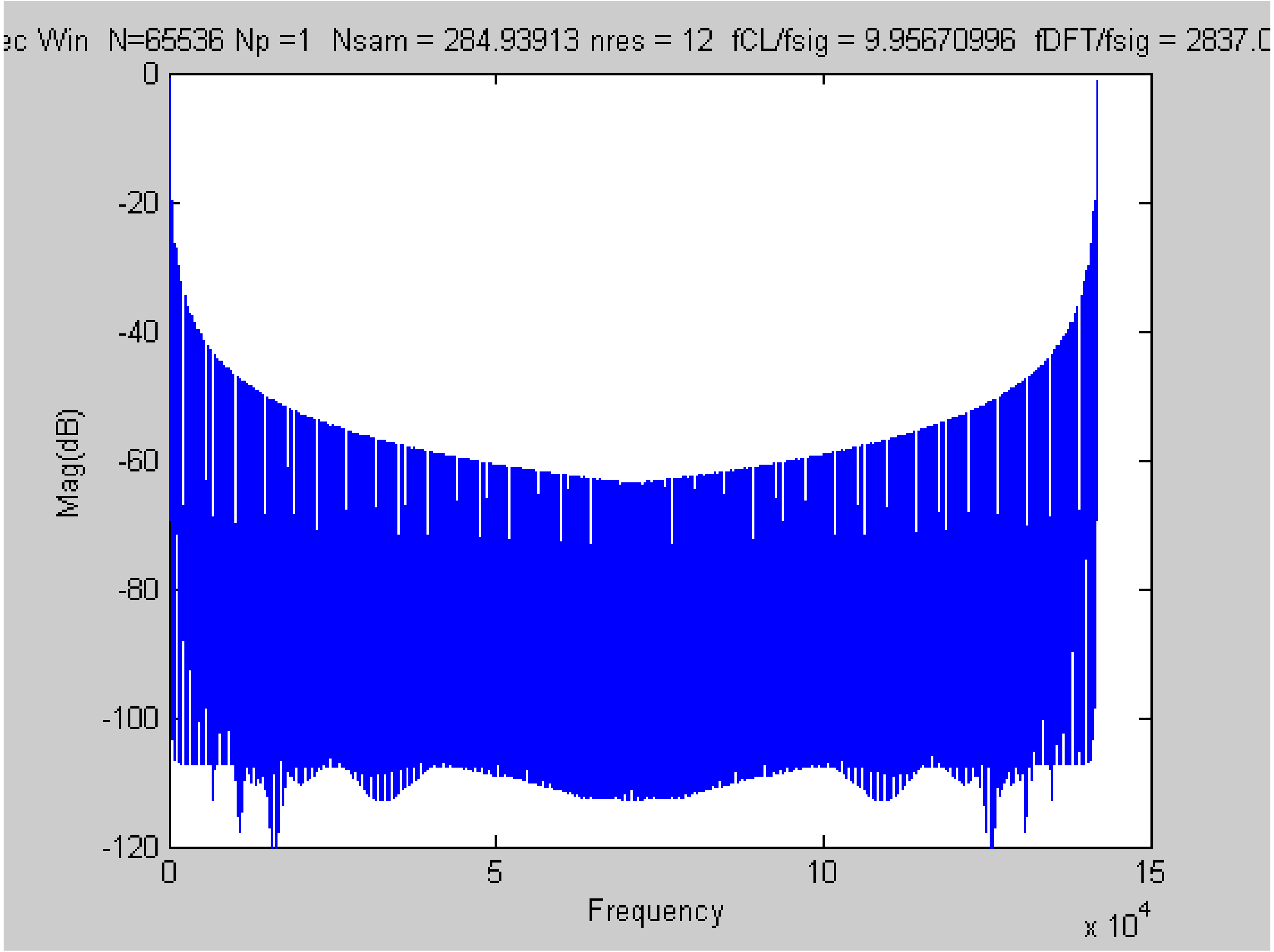
Spectral Characteristics of DAC

Consider the following example

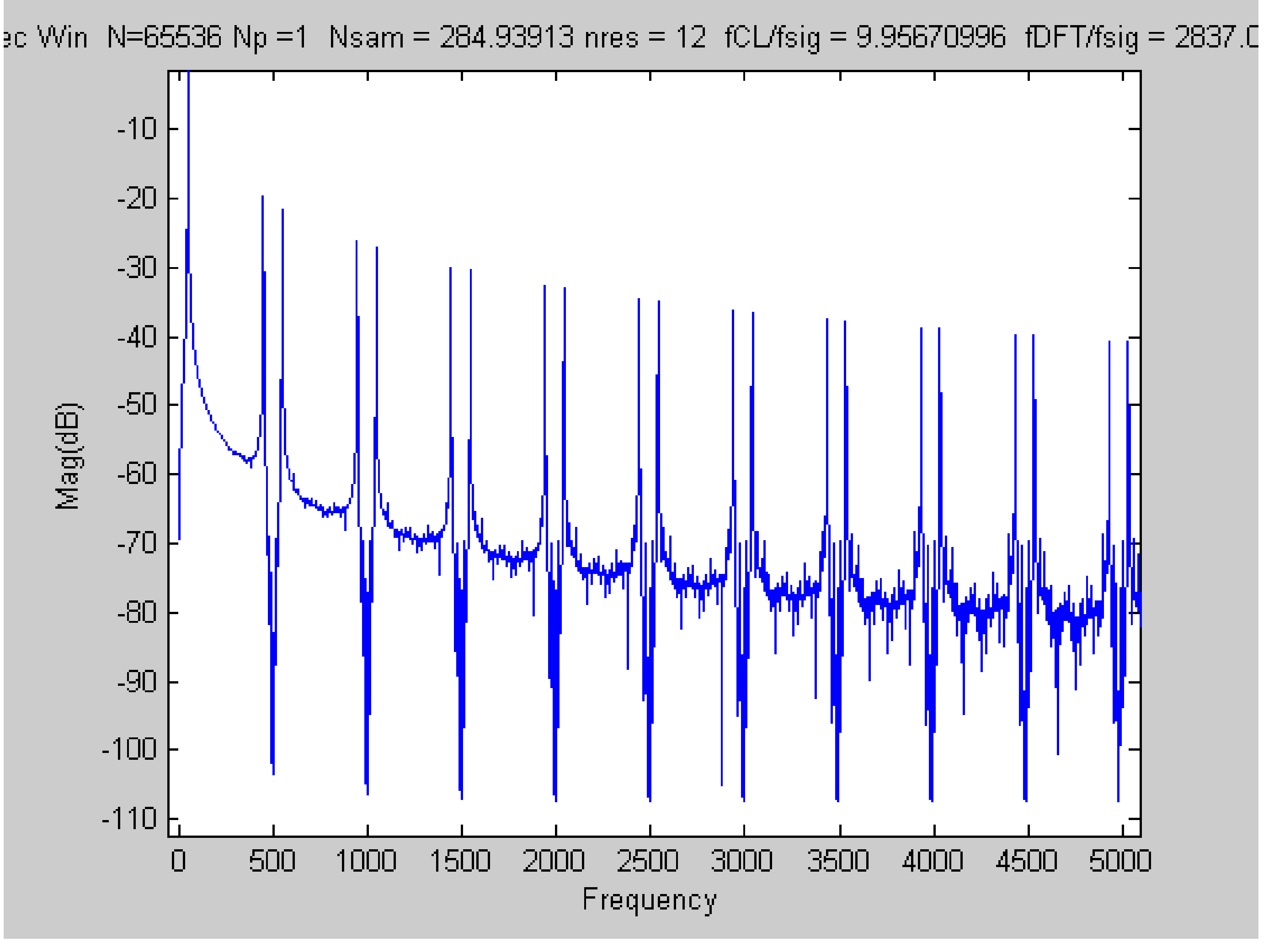
- f_{SIG}=50 Hz
- f_{CL}=497.8 Hz (DAC clock)
- f_{DFTCL}=141.853K Hz (not coherent sampling)
- n_{DFT}=16
- N_{P1}=23 (number of signal periods in DFT window)
- N_P=1
- n_{res}=16bits
- $Xin(t) = .95\sin(2\pi f_{SIG} t)$ (-.4455dB)

Matlab File: afft_Quantization_DAC.m

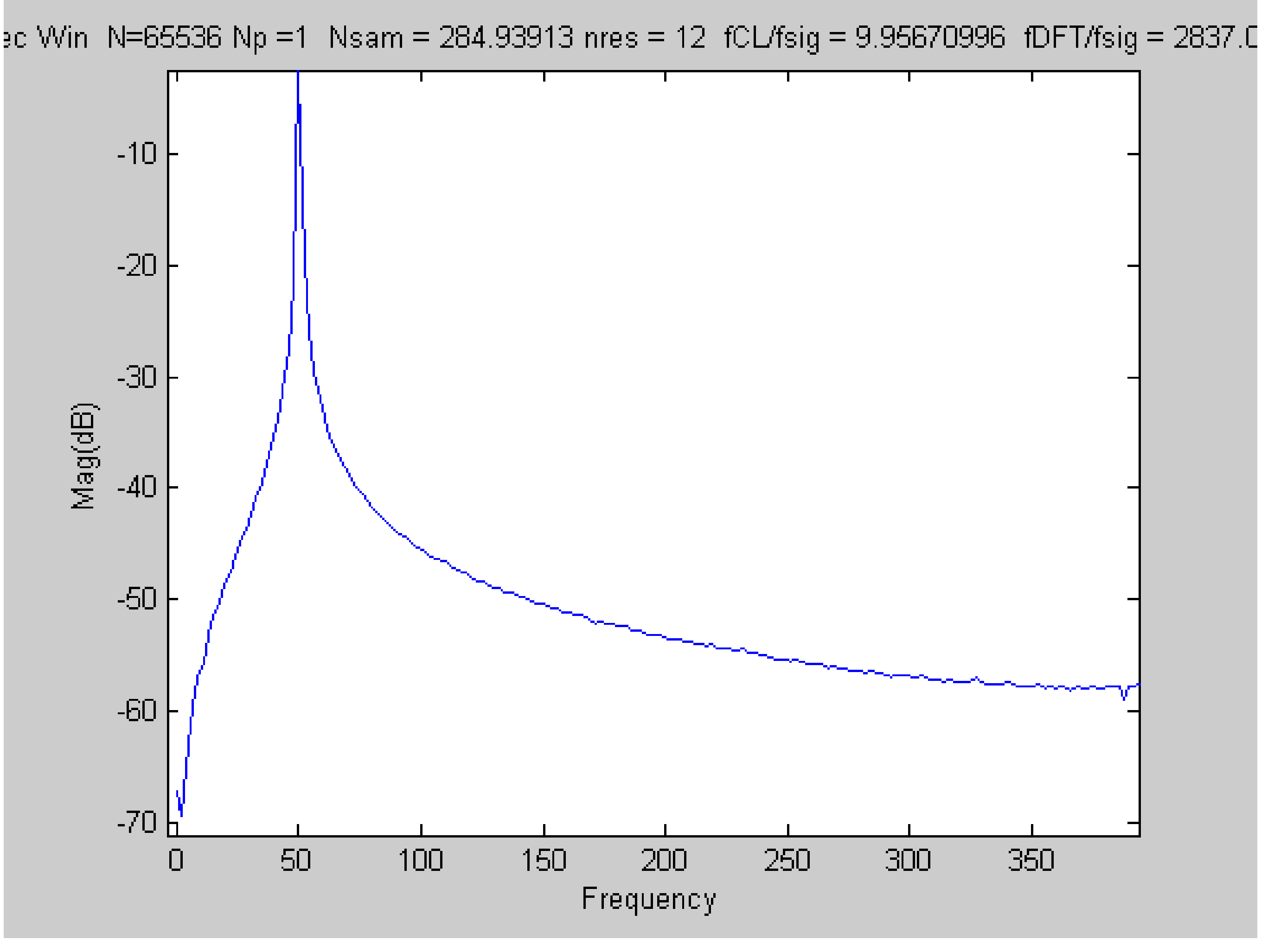
DFT Simulation from Matlab



DFT Simulation from Matlab



DFT Simulation from Matlab



Summary of time and amplitude quantization assessment

Time and amplitude quantization do not introduce harmonic distortion

Time and amplitude quantization do increase the noise floor

Stay Safe and Stay Healthy !

End of Lecture 7